Seeking multiple solutions: multimodal optimization using niching methods

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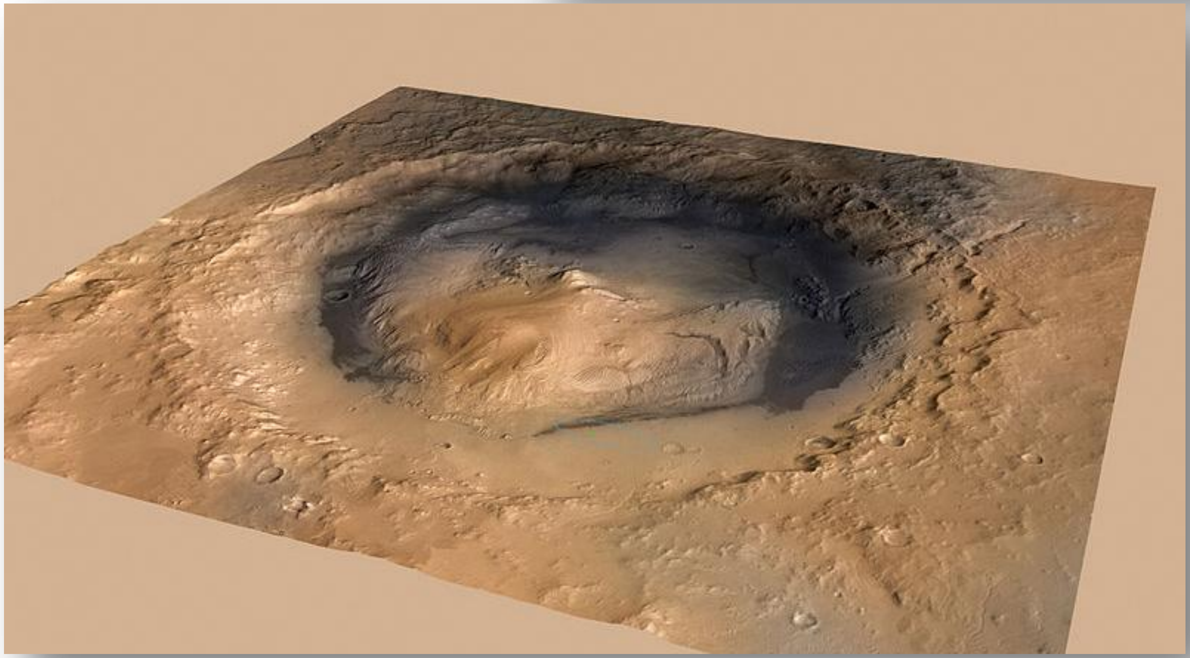
Multi-Modal Optimization?

Curiosity's view of "Mount Sharp" (September 9, 2015)



Mount Sharp rises from the middle of Gale Crater; the green dot marks Curiosity's landing site (north is down).

Source: https://en.wikipedia.org/wiki/Curiosity_(rover)

Outline

- Background on Multi-Modal Optimization (**MMO**), i.e., finding multiple good solutions, in meta-heuristics
- What are the benefits for studying **MMO**?
- What are niching methods?
- Some real-world examples
- Classic niching methods: fitness sharing, crowding, clearing, and speciation, etc.
- PSO and DE for niching methods
- Niching in dynamic and multiobjective optimization
- IEEE CIS Taskforce on Multi-Modal Optimization
- Summary
- References

What is multimodal optimization?

- **Multi-Modal Optimization (MMO)**: to locate multiple optimal (or close to optimal) solutions in the search space.
- This is different from a conventional optimization method which has a common goal of seeking to locate a single global optimum.
- **MMO** problems represent an important class of optimization problems. Many real-world optimization problems are multimodal by nature, that is, multiple satisfactory solutions exist (several real-world examples of MMO problems are provided in subsequent slides).
- From a decision maker's point of view, it might be desirable to locate all global optima and/or some local optima that are also considered as being satisfactory.

Methods for MMO

- Optimization methods specifically designed for solving MMO problems, often called multimodal optimization or niching methods, are predominately developed from the field of **meta-heuristic algorithms**, which covers a family of population-based stochastic optimization algorithms, including evolutionary algorithms, evolutionary strategies, particle swarm optimization, differential evolution, and so on.
- These meta-heuristic algorithms are shown particularly effective in solving multimodal optimization problems, if equipped with specifically designed diversity preserving mechanisms, commonly referred to as **niching methods**.

What are the benefits?

- A decision maker may be interested to know whether there exist multiple equally good solutions before making a final decision.
- Important for a sensitivity study of a problem, and helps develop more **robust solutions** to the problem.
- Plays an important role in keeping a diverse population of candidate solutions, hence helps prevent the population from converging prematurely to a sub-optimum.
- May increase the probability of finding the global optimum.

Ecological inspiration

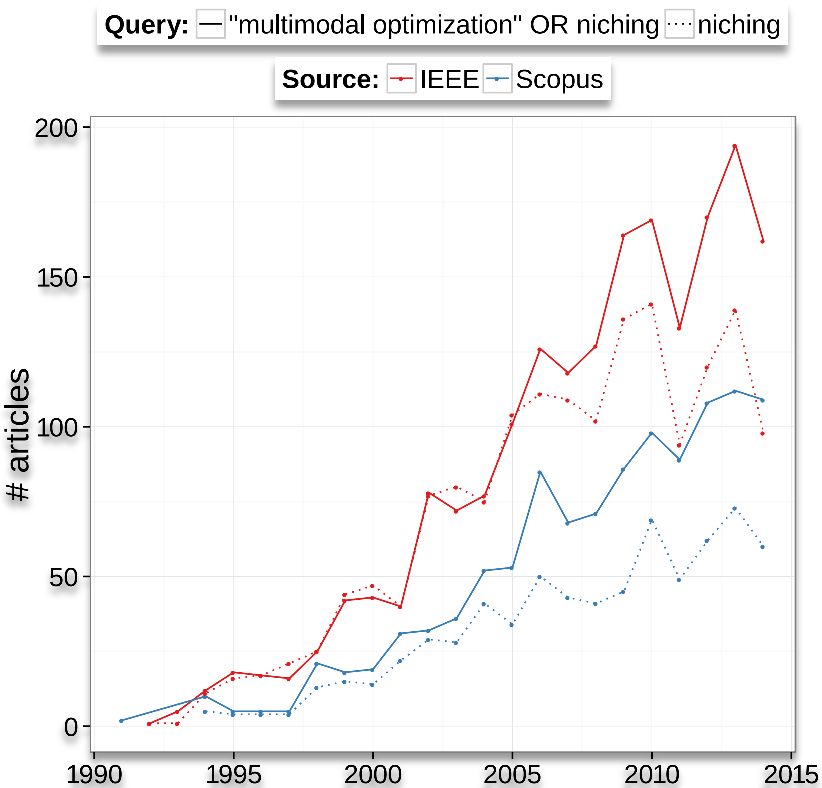
- In natural ecosystems, individual species must compete to survive by taking on different roles. Different species evolve to fill different niches (or subspaces) in the environment that can support different types of life.

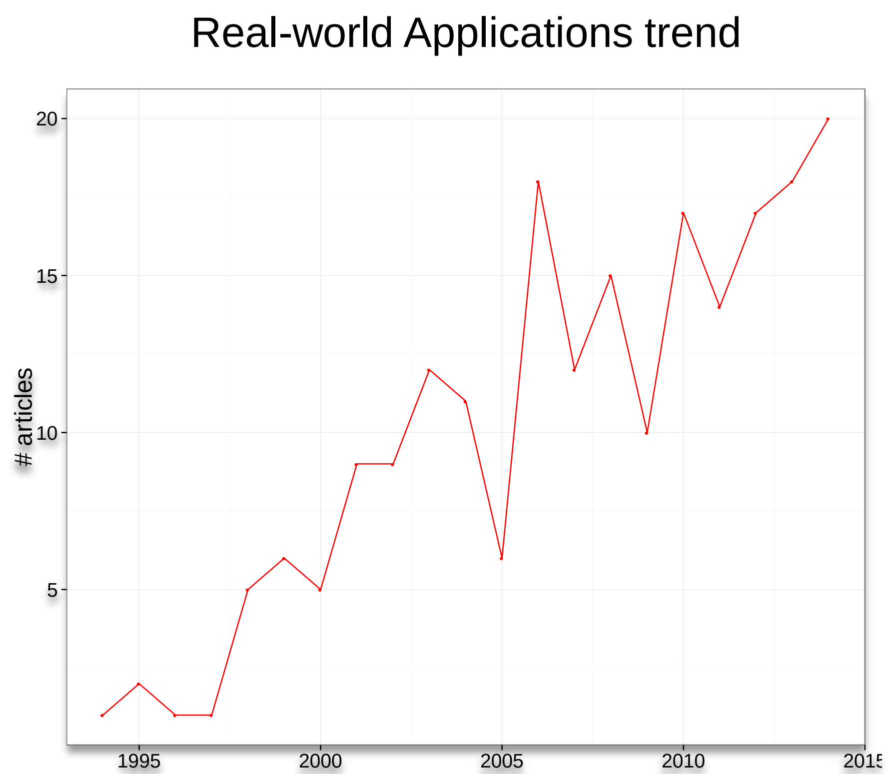
What are niching methods?

- According to the Oxford Dictionary, a **niche** refers to "*a role taken by a type of organism within its community*"; and a **species** refers to "*a group of living organisms consisting of similar individuals capable of exchanging genes or interbreeding*".
- These concepts of niches, species and speciation can be adopted in a **population-based meta-heuristic algorithm (typically an evolutionary algorithm)**, to encourage the population to evolve different species targeting different optimal solutions in the search space.

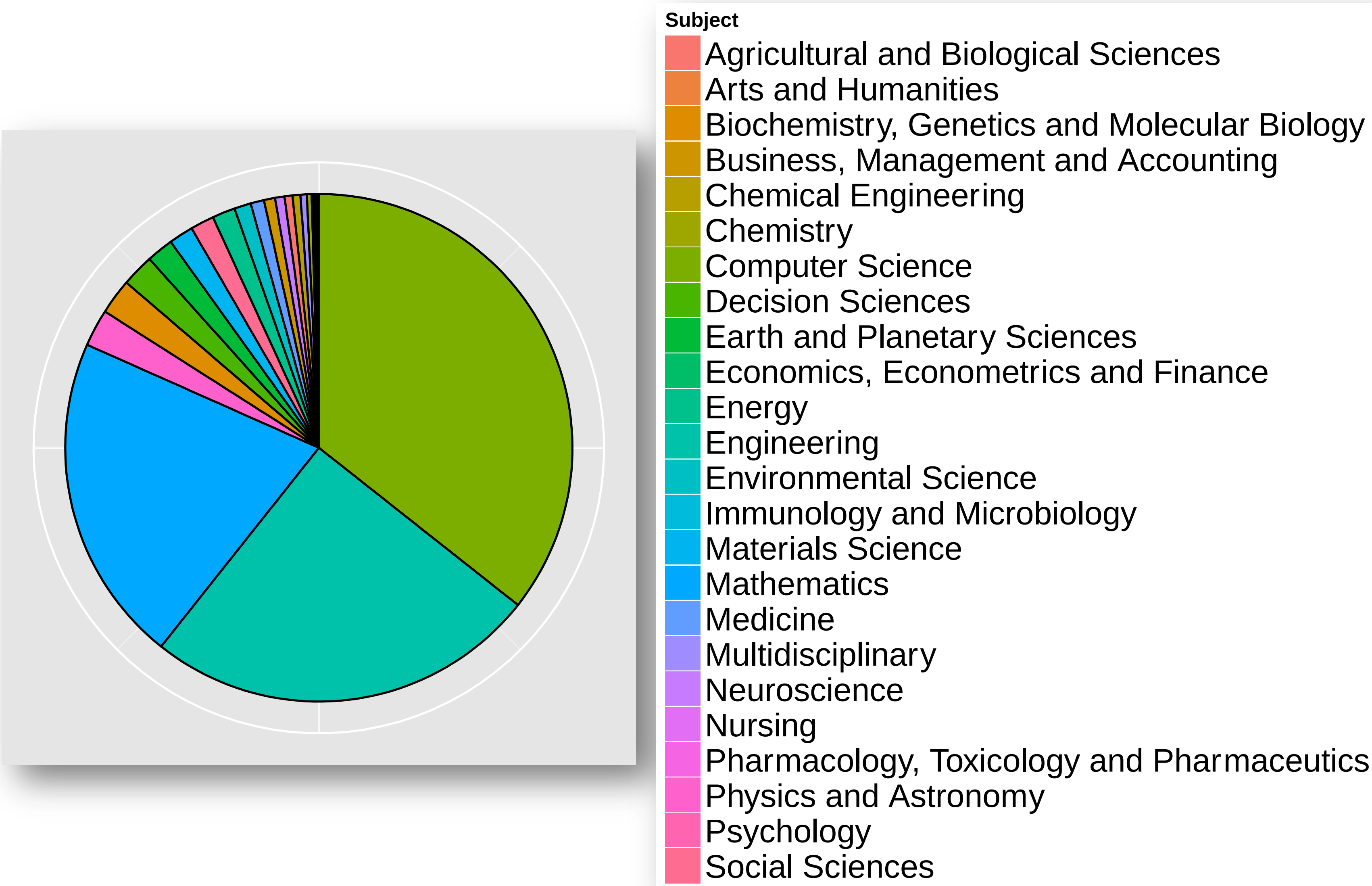
MMO Publication trends

- Despite niching methods first appeared more than 30 years ago, currently niching techniques are experiencing a revival, attracting researchers from across a wide range of research fields





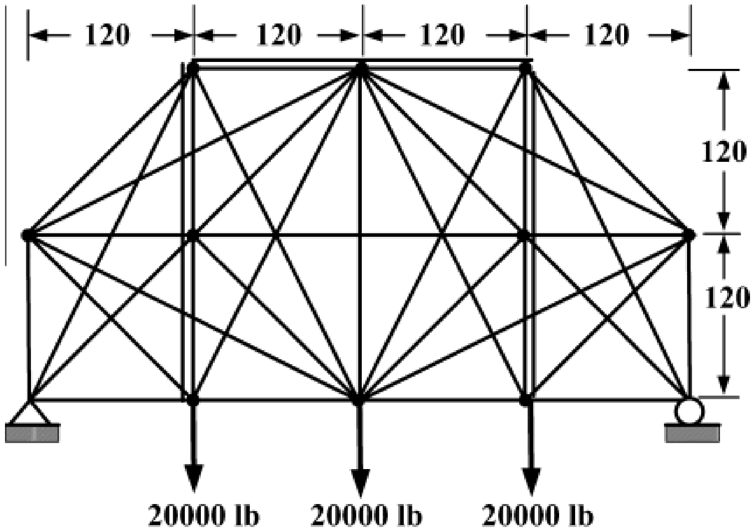
MMO Application areas



Engineering example: truss topology design

- In topology optimization, the connectivity of members in a truss is to be determined. There exist multiple different topologies with almost equal overall weight in truss-structure design problems as the members in the ground structure increase .
- The resulting solution of truss-structure optimization design problems becomes ‘‘**multi-modal**’’ with large number of truss members.

Some nodes in the **ground structure** may or may not be removed. The optimal structure is found as a subset of the ground structure.



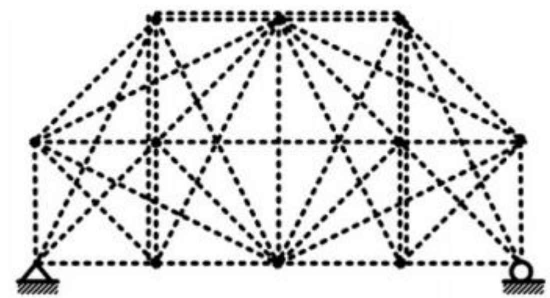
Deb K, Gulati S. “Design of truss-structures for minimum weight using genetic algorithms,” *Finite Elements Anal Des* 2001;37:447–65.

G.-C. Luh and C.-Y. Lin, “Optimal design of truss-structures using particle swarm optimization,” *Computers and Structures*, vol. 89, no.23-24, pp. 2221 – 2232, Dec. 2011.

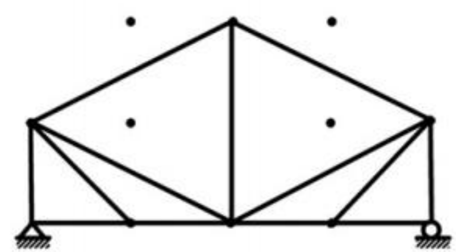
Truss topology design

- Sharing scheme is used to compute the similarity between different topology design solutions.
- The sharing fitness is a reduced one from the original fitness, in order to discourage solutions in the vicinity.

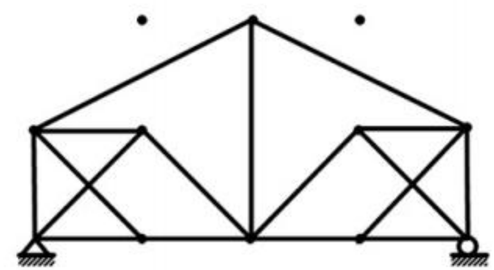
Binary PSO is run based on the sharing fitness values, and multiple dissimilar truss topologies are derived and saved.



(*a*) Two-tier, 39-member, 12-node ground structure

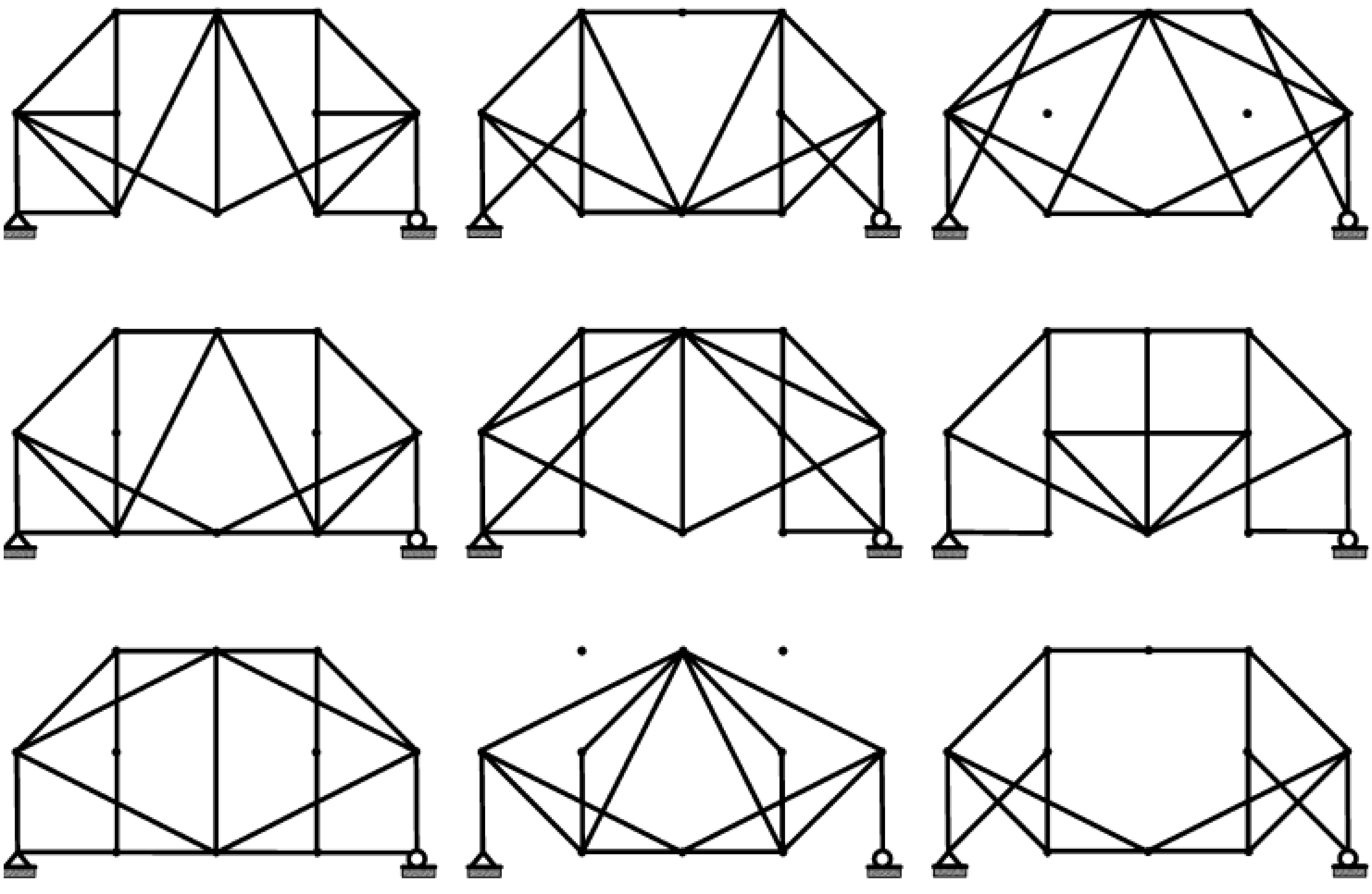


(*b*) Two-tier, 13-member, 8-node truss



(*c*) Two-tier, 17-member, 10-node truss

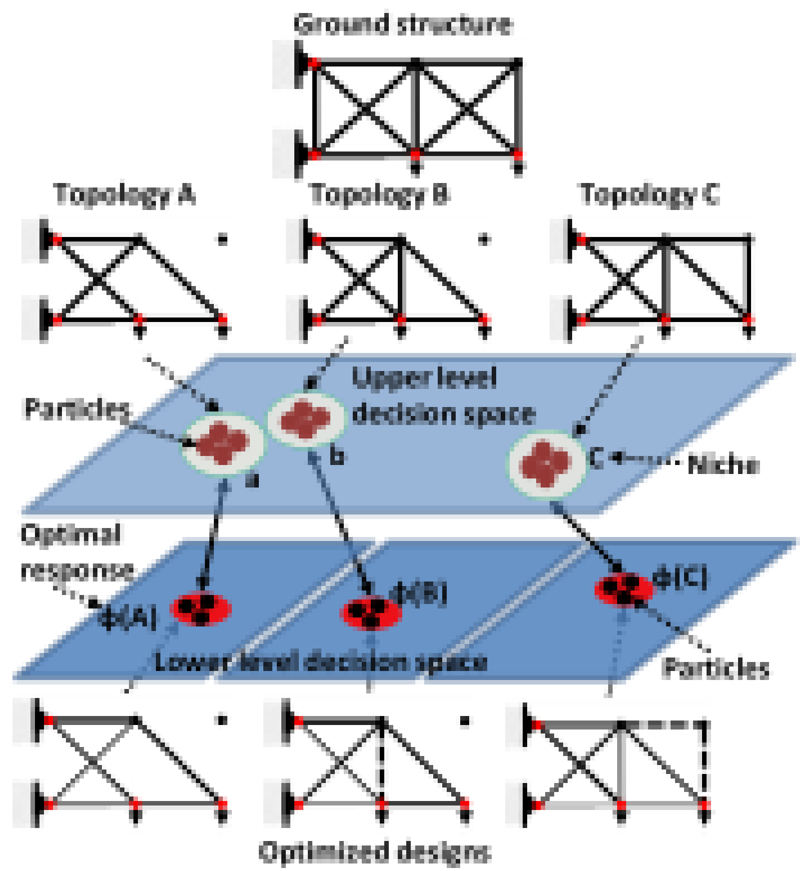
Truss topology design



Multiple optimal truss topologies found by BPSO with niching.

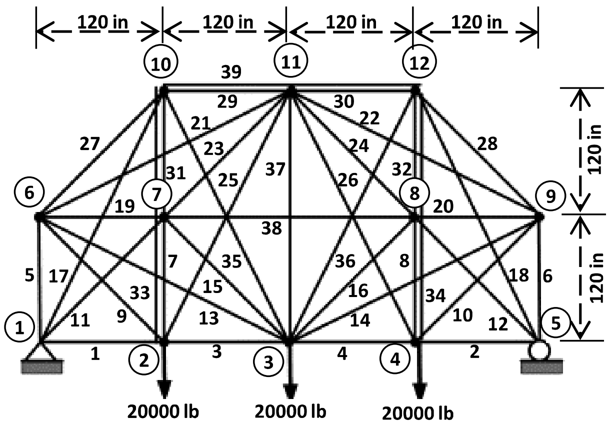
Trust structure design using Bilevel and niching aspects

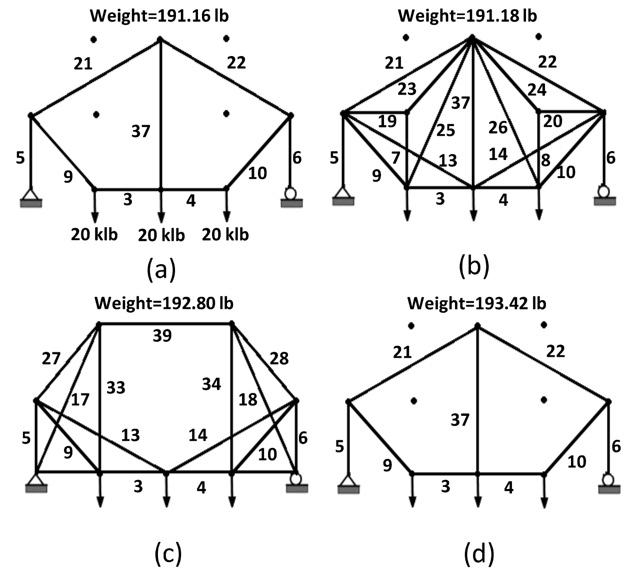
- Formulate the truss problem as a **bilevel** optimization problem
- A new bilevel PSO niching method locates multiple optimal solutions
- **Stable topologies** can be found in the **upper level**
- The **optimized sizes** of the members of these topologies can be found in the **lower level**
- **Niching** at the **upper level**
- **Standard optimizer** is used at the **lower level** to optimize a bilevel truss problem

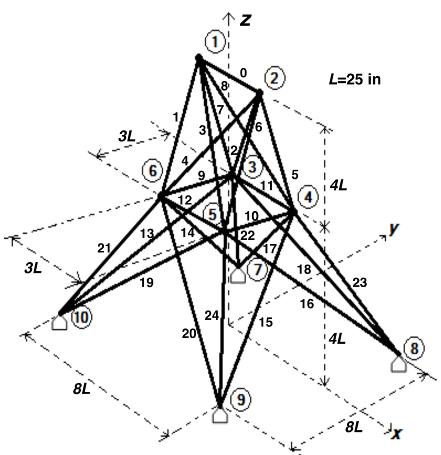


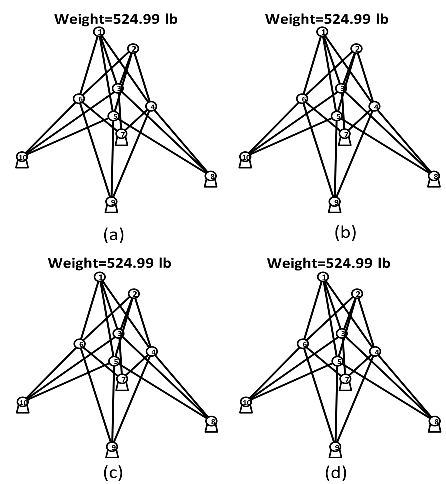
Md. Jakirul Islam, Xiaodong Li, and Kalyanmoy Deb. 2017. Multimodal Truss Structure Design Using Bilevel and Niching Based Evolutionary Algorithms. In Proceedings of GECCO ’17, Berlin, Germany, July 15-19, 2017, DOI: http://dx.doi.org/10.1145/3071178.3071251

Trust structure design examples



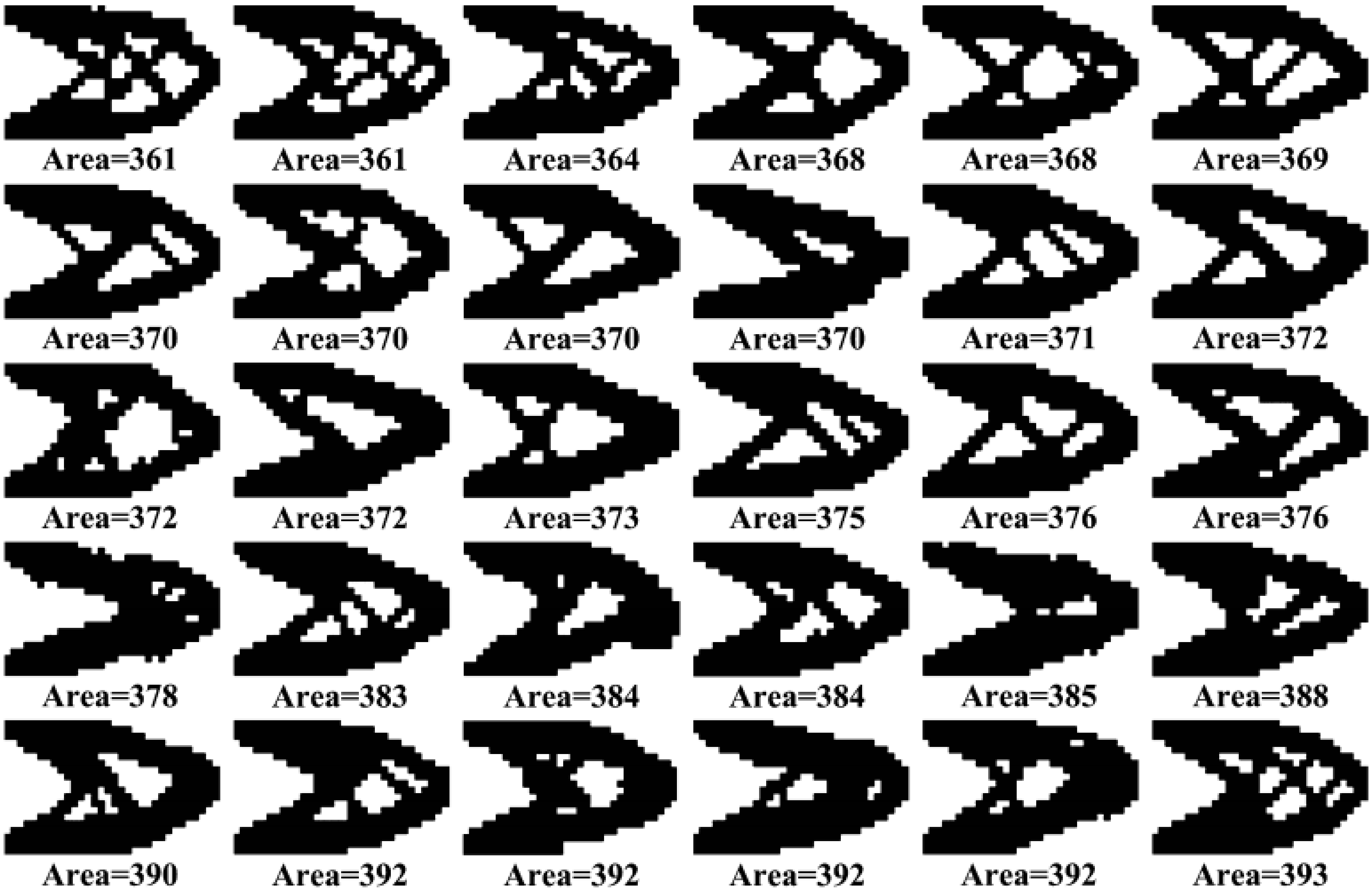






Md. Jakirul Islam, Xiaodong Li, and Kalyanmoy Deb. 2017. Multimodal Truss Structure Design Using Bilevel and Niching Based Evolutionary Algorithms. In Proceedings of GECCO '17, Berlin, Germany, July 15-19, 2017, DOI: http://dx.doi.org/10.1145/3071178.3071251

Continuum structural topology optimization



G.-C. Luh, C.-Y. Lin, Y.-S. Lin, “A binary particle swarm optimization for continuum structural topology optimization”, *Applied Soft Computing*, Volume 11, Issue 2, March 2011, Pages 2833-2844, ISSN 1568-4946,

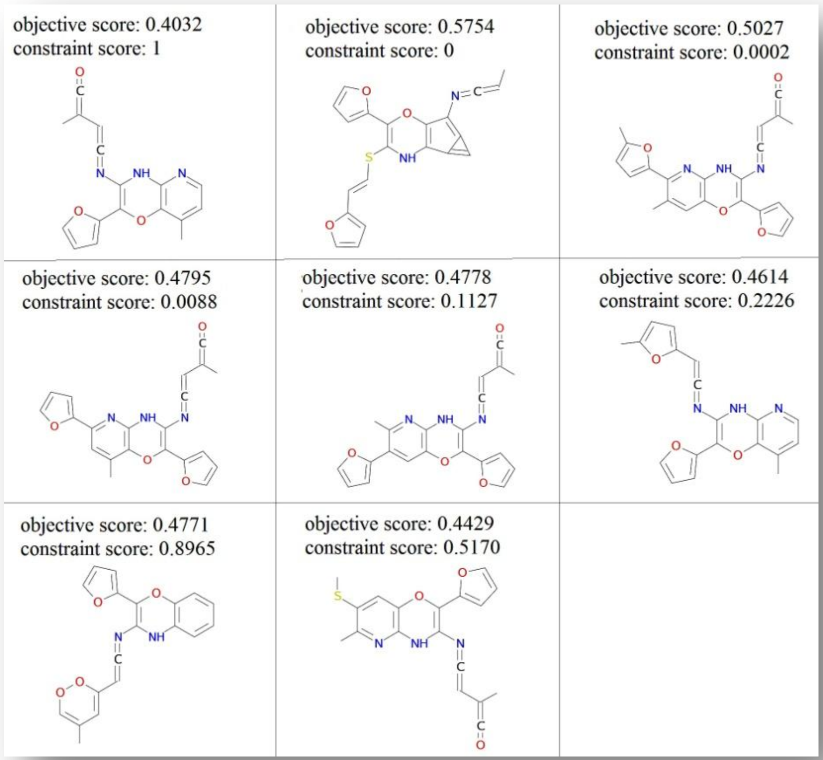
Drug Molecule Design (I)

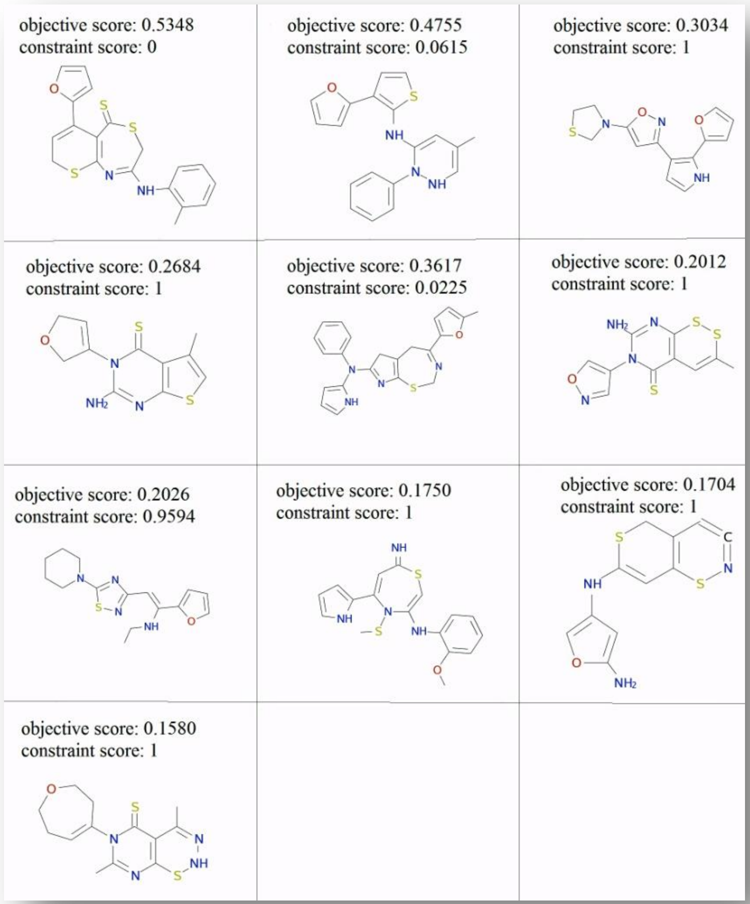
- Search for **molecular structures** with specific pharmacological or biological activity that influence the behavior of certain targeted cells
- **Objectives**: Maximization of potency of drug & Minimization of side-effects
- **Aim:** provide the medicinal chemist a **set of diverse molecular structures** that can be promising candidates for further research
  - **Fit solutions may** result in finding structures that **fail in practice**
  - The chemist **desires a set of promising structures** rather than only one single solution

J. W. Kruisselbrink, A. Aleman, M. T. M. Emmerich, A. P. Ijzerman, A. Bender, T. Baeck, and E. van der Horst, “Enhancing search space diversity in multi-objective evolutionary drug molecule design using niching,” GECCO’09, 2009, pp. 217–224.

Drug Molecule Design (II)

- Dynamic Niche Sharing technique incorporated to MOEA



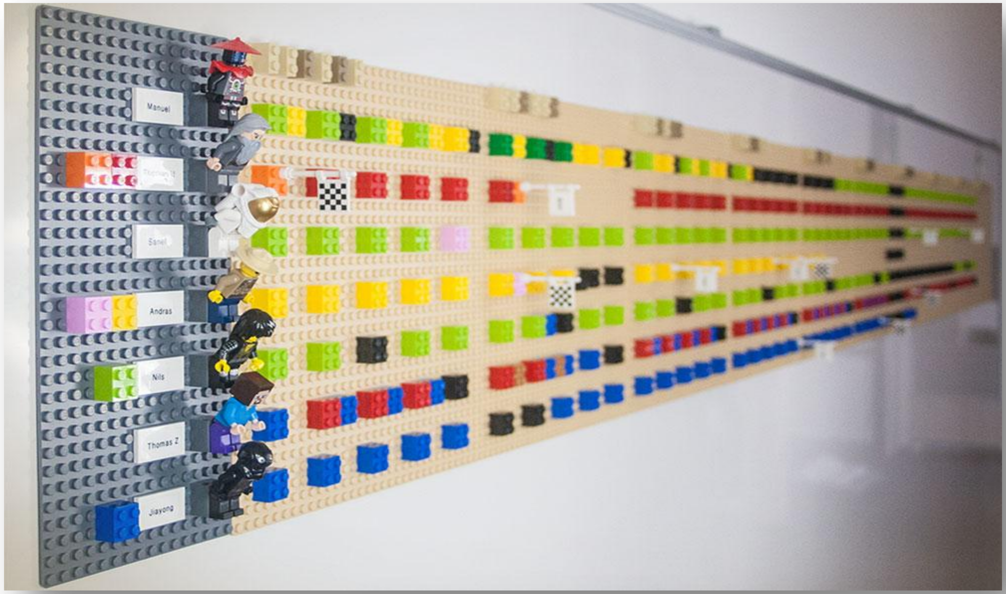


Without (left) and with (right) niching

J. W. Kruisselbrink, A. Aleman, M. T. M. Emmerich, A. P. Ijzerman, A. Bender, T. Baeck, and E. van der Horst, “Enhancing search space diversity in multi-objective evolutionary drug molecule design using niching,” GECCO’09, 2009, pp. 217–224.

Scheduling Problems

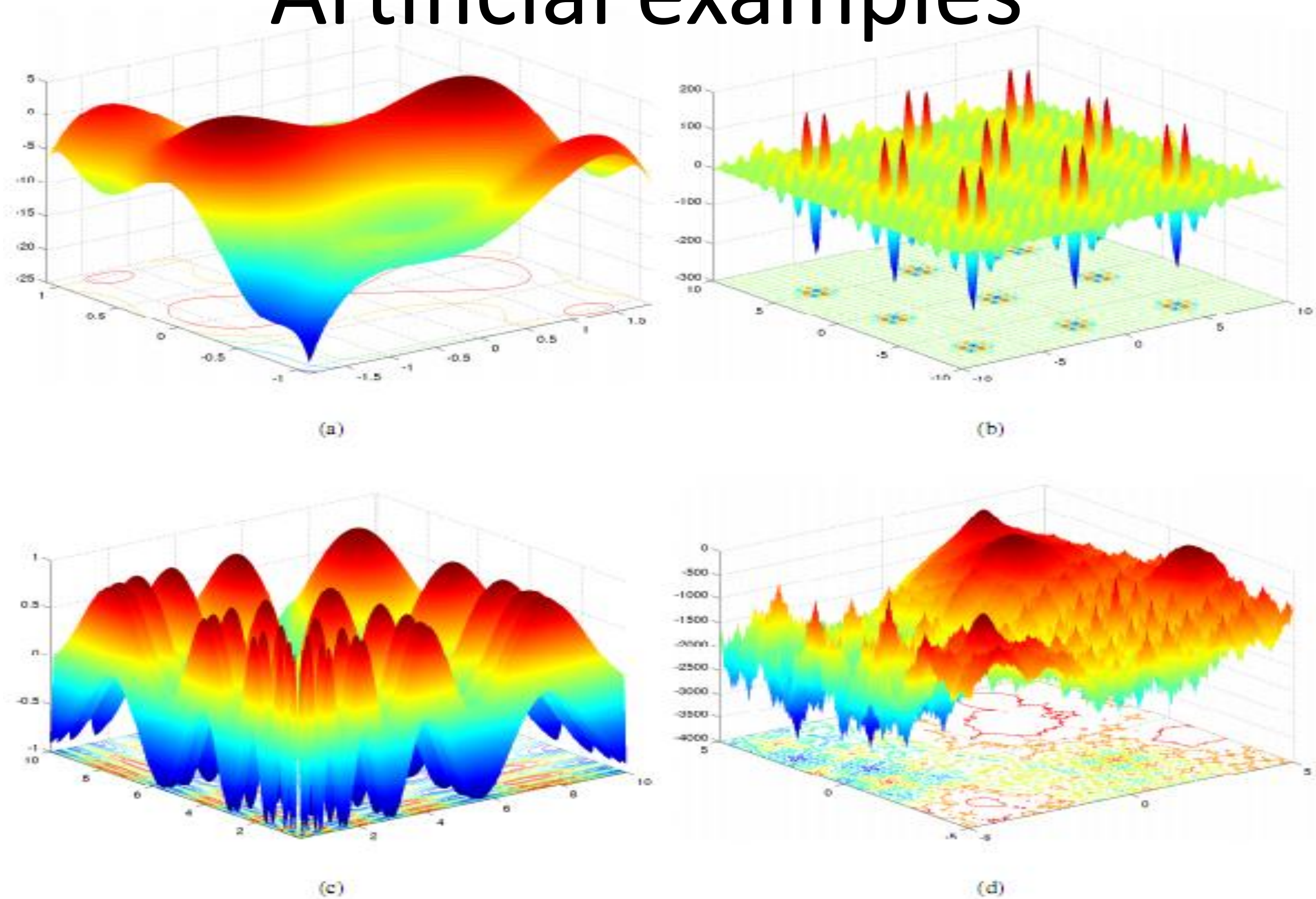
- Project Management
  - Optimize productivity
    - Makespan, Due dates
  - Maximize revenue
  - Minimize delays
- Job shop Scheduling



- Pérez, E., Posada, M. & Lorenzana, A. Taking advantage of solving the resource constrained multi-project scheduling problems using multi-modal genetic algorithms, Soft Comput (2016) 20: 1879. doi:10.1007/s00500-015-1610-z
- E. Prez, F. Herrera, and C. Hernndez, "Finding multiple solutions in job shop scheduling by niching genetic algorithms," Journal of Intelligent Manufacturing, vol. 14, no. 3-4, pp. 323–339, 2003.
- E. Prez, M. Posada, and F. Herrera, "Analysis of new niching genetic algorithms for finding multiple solutions in the job shop scheduling," Journal of Intelligent Manufacturing, vol. 23, no. 3, pp. 341–356, 2012.

Pictures from: http://www.ymc.ch/en/lego-resource-scheduling-wall

Artificial examples



(a) (b) (c) (d)

X. Li, A. Engelbrecht, and M. Epitropakis, “Benchmark functions for cec’2013 special session and competition on niching methods for multimodal function optimization,” Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, 2013.

Demos

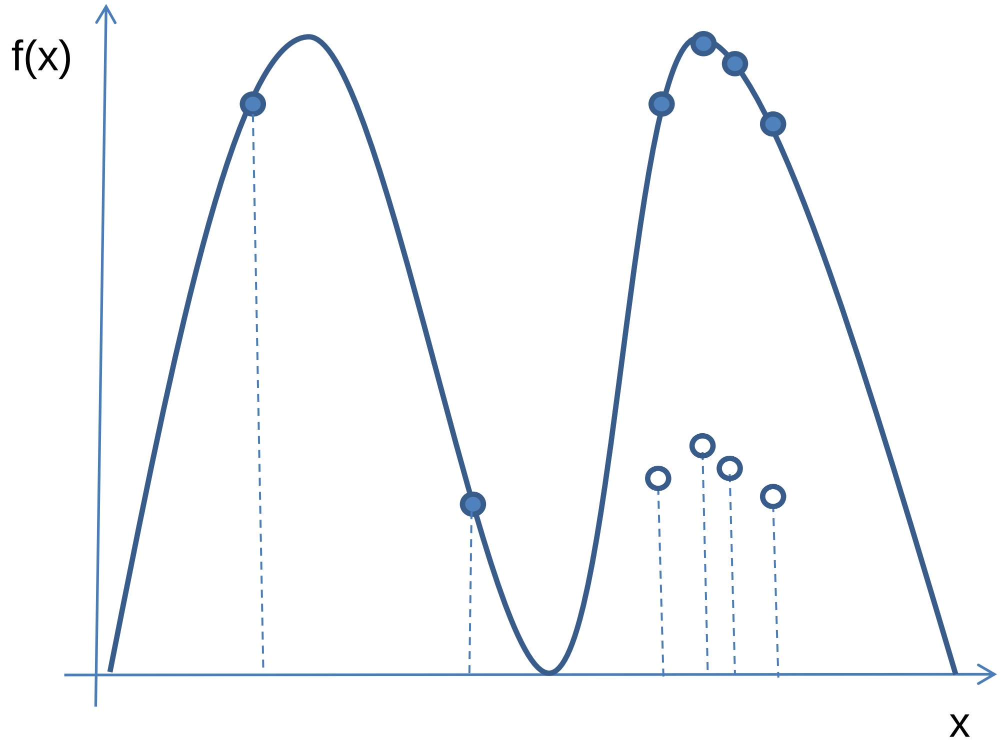
- DE/nrand/1 on Himmelblau 2D: https://www.youtube.com/watch?v=M32JdyBmVLc
- DE/nrand/1 for Shubert 2D: https://www.youtube.com/watch?v=miy3VK_8KwU
- DEnrand on Deb's function with 100 optima 500 pop size: https://www.youtube.com/watch?v=ymNRKLrSyQU

Classic niching methods

Fitness sharing

- A sharing function can be used to degrade an individual's fitness based on the presence of other neighbouring individuals.

- During selection, many individuals in the same neighbourhood would degrade each other's fitness, thereby limiting the number of individuals occupying the same niche.



An example to illustrate fitness sharing.

D. E. Goldberg and J. Richardson, "Genetic algorithms with sharing for multimodal function optimization," in Proc. of the Second International Conference on Genetic Algorithms, J. Grefenstette, Ed., 1987, pp. 41–49.

Crowding methods

- Originally by De Jong (1975), and later modified by Mahfoud (1995).
- **Crowding** usually consists of two phases:
  - Pairing phase: pairing each offspring with a similar individual in the current population; and
  - Replacement phase: which of the two will remain in the population?
- **Deterministic Crowding** selects the fittest individual in each pair in the replacement phase. Probabilistic Crowding selects the surviving individual for each pair based on a probabilistic formula that takes fitness into account.

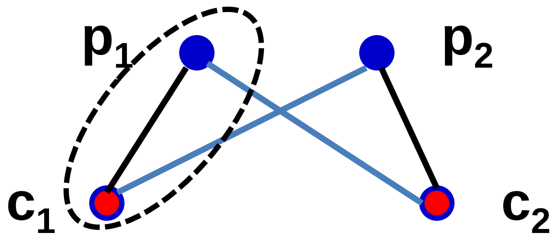
S. W. Mahfoud. *Niching Methods for Genetic Algorithms*. PhD thesis, Department of General Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, 1995.

K. A. de Jong. *An Analysis of the Behavior of a Class of Genetic Adaptive Systems*. PhD thesis, Department of Computer and Communication Sciences, University of Michigan, Ann Arbor, MI, 1975.

Deterministic crowding

Algorithm 1: The pseudocode of deterministic crowding.

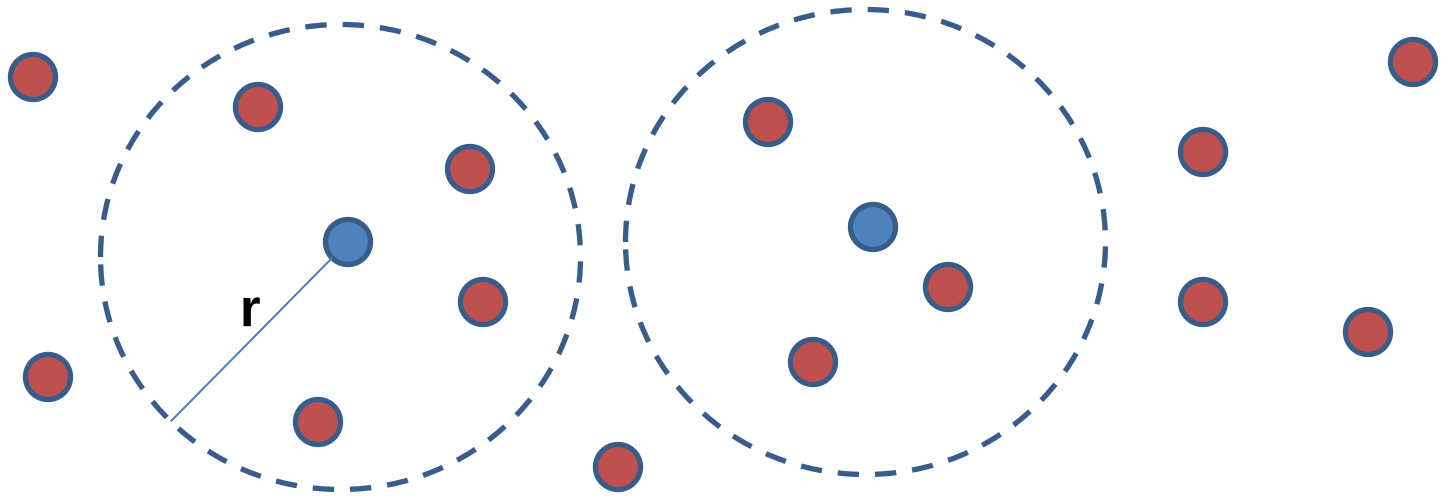
1: Select two parents, p_1 and p_2 randomly, without replacement
2: Generate two offspring c_1 and c_2
3: **if** $d(p_1, c_1) + d(p_2, c_2) \leq d(p_1, c_2) + d(p_2, c_1)$ **then**
4:     **if** $f(c_1) > f(p_1)$ **then** replace p_1 with c_1
5:     **if** $f(c_2) > f(p_2)$ **then** replace p_2 with c_2
6: **else**
7:     **if** $f(c_2) > f(p_1)$ **then** replace p_1 with c_2
8:     **if** $f(c_1) > f(p_2)$ **then** replace p_2 with c_1
9: **end if**



Each offspring tends to compete for survival with its most similar parent.

Clearing

- Proposed by Petrowski (1996); inspired by the principle of sharing of limited resources within each subpopulation (or species). The clearing procedure only supplies the resources to the best individuals in each subpopulation.
- All individuals fall within **r** distance from the best **k** individuals (below shows **k** = 2) from the population are **cleared**. This process is repeated until the whole population is considered.



A. Petrowski. A clearing procedure as a niching method for genetic algorithms. In *Proceedings of Third IEEE International Conference on Evolutionary Computation*(ICEC'96), pages 798–803. Piscataway, NJ:IEEE Press, 1996.

Niching with PSO and DE

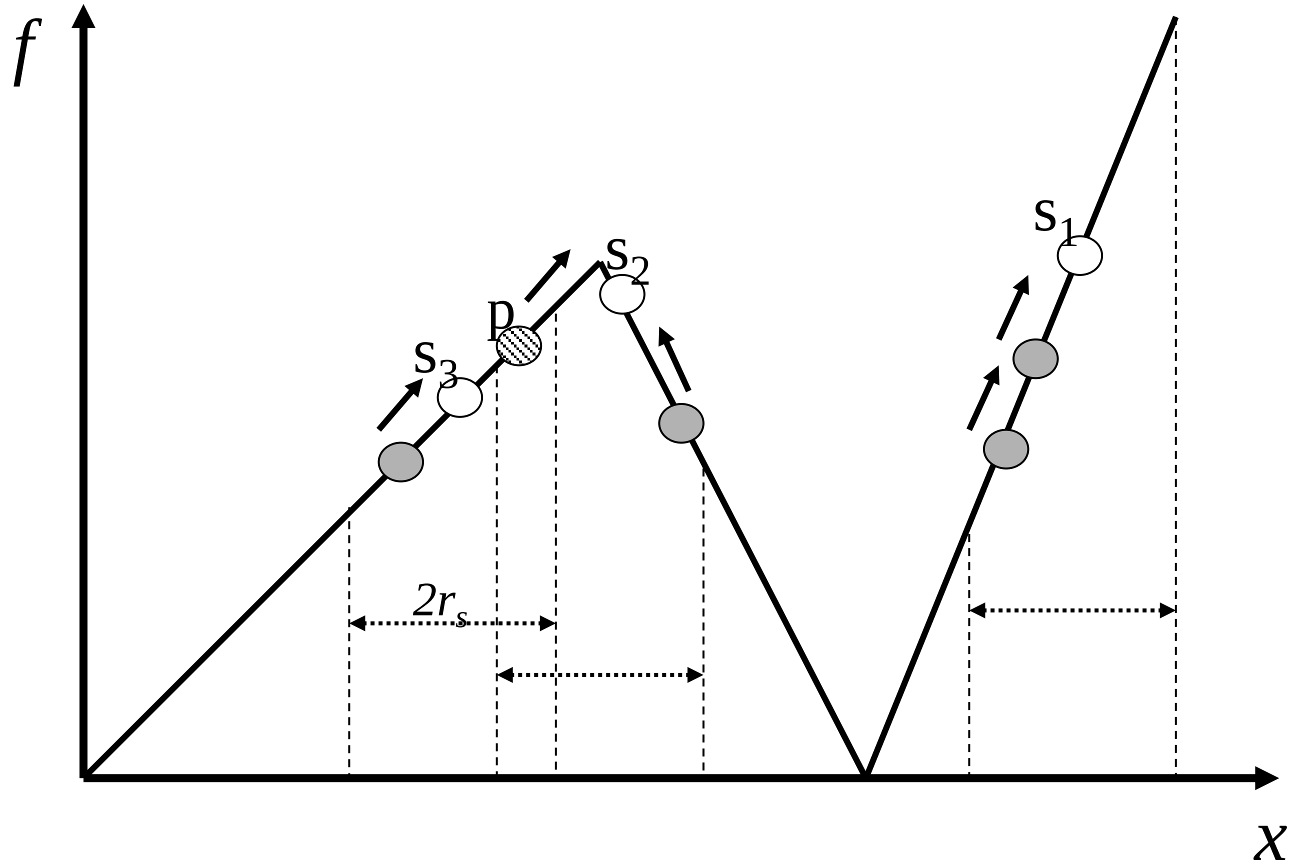
PSO niching methods

- In particle swarm optimization (PSO), each particle has its own memory remembering its best known position so far, and share this information with other particles.
- At each iteration, each particle is propelled towards the area defined by the stochastic average of its own known best position and the swarm best position.
- The notion of memory associated with each particle is unique to PSO, and this property can be used to induce niching behaviour: a swarm can be divided into two parts, an **explorer-swarm** consisting of the current particles, and a **memory-swarm**, comprising of only best known positions of individual particles.

X. Li, “Developing niching algorithms in particle swarm optimization,” in *Handbook of Swarm Intelligence*, ser. Adaptation, Learning, and Optimization, B. Panigrahi, Y. Shi, and M.-H. Lim, Eds. Springer Berlin Heidelberg, 2011, vol. 8, pp. 67–88.

Speciation-based PSO

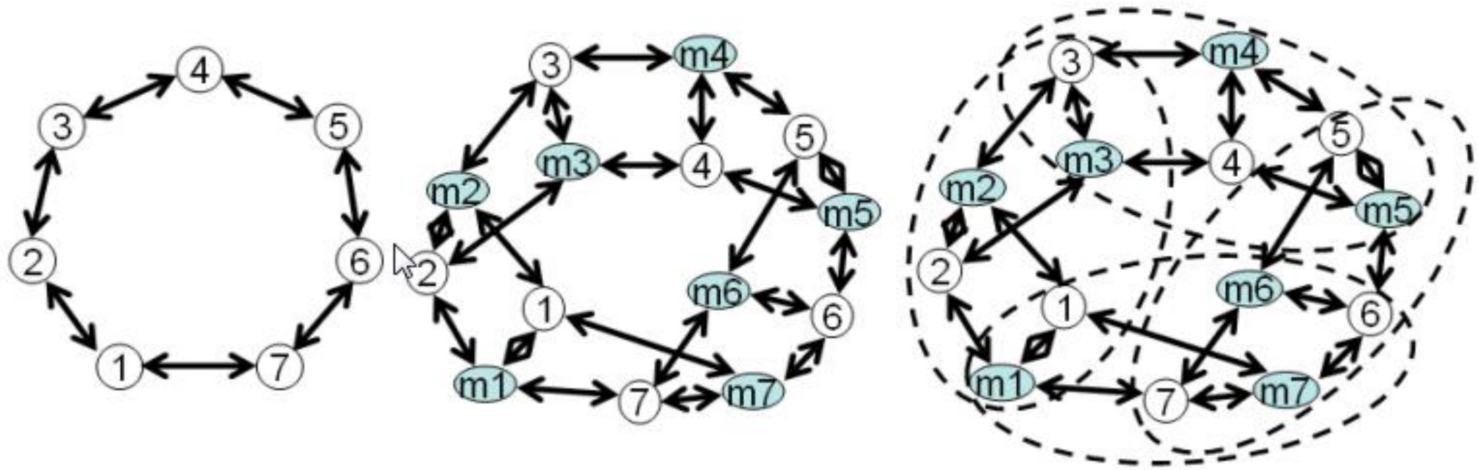
An example of how to determine the species seeds from the population at each iteration. s_1, s_2, and s_3 are chosen as the species seeds. Note that p follows s_2.



D. Parrott and X. Li, "Locating and tracking multiple dynamic optima by a particle swarm model using speciation," IEEE Trans. on Evol. Comput., vol. 10, no. 4, pp. 440–458, August 2006.

Ring topology based niching PSO

- Given a reasonably large population uniformly distributed in the search space, the ring topology based niching PSOs are able to form stable niches across different local neighbourhoods, eventually locating multiple global/local optima.
- This method can operate as a niching algorithm by using individual particles' local memories to form a stable network retaining the best positions found so far,



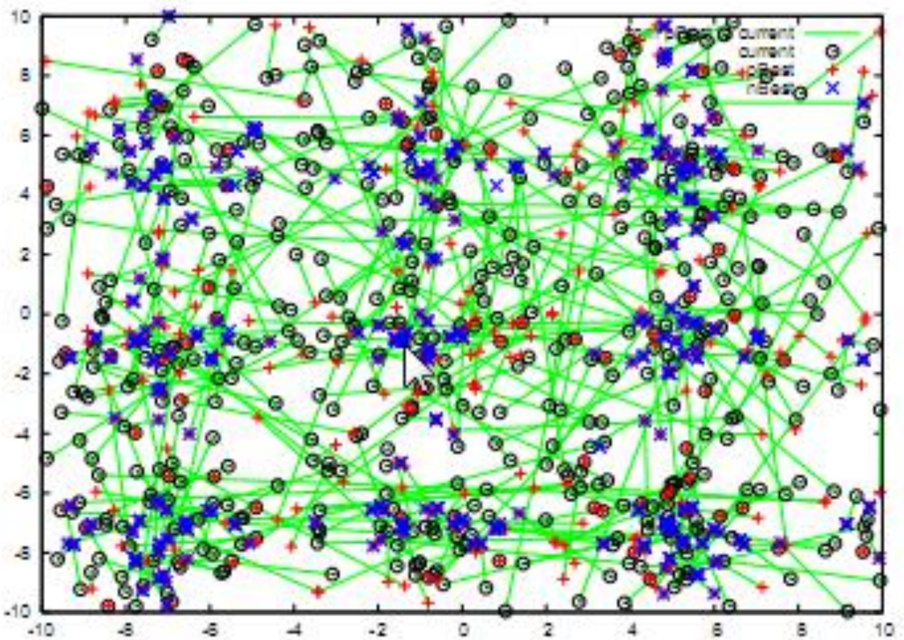
a) b) c)

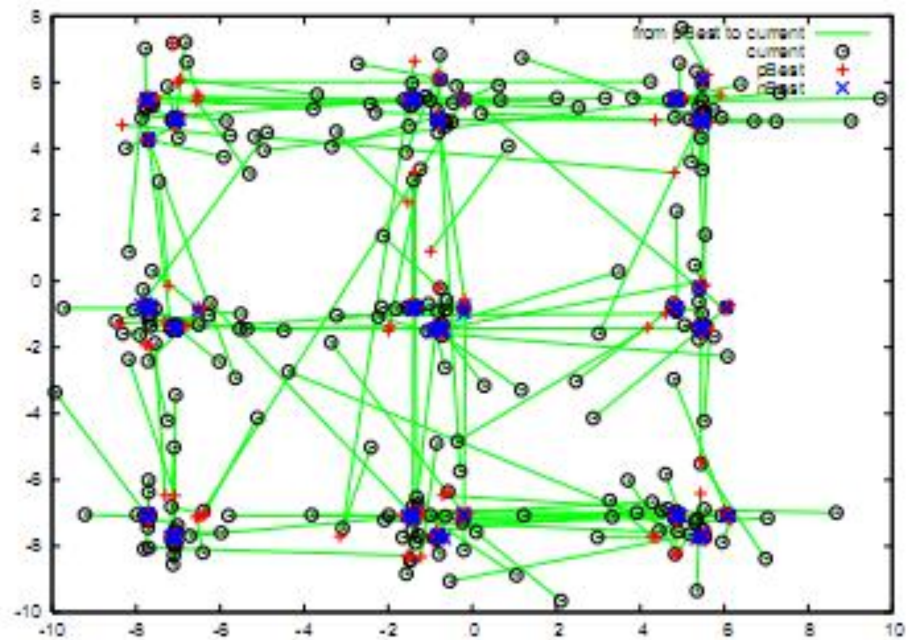
X. Li, "Niching without niching parameters: Particle swarm optimization using a ring topology," *IEEE Trans. on Evol. Comput.*, vol. 14, no. 1, pp. 150 – 169, February 2010.

RMIT University

Ring topology based niching PSO

- Results on Shubert 2D function (two snapshots during a simulation run).





DE niching methods

- Studies on the dynamics of DE suggest that the DE individuals are inclined to cluster around either local or global optima after some numbers of iterations.
- Inspired by this observation, the DE mutation operator in a classic DE variant DE/nrand/1, was altered to incorporate the nearest neighbour concept, in order to induce the niching effect.
- Instead of using the base vector the usual way, its nearest neighbour is always chosen as the actual base vector.

1) **"DE/nrand/1"**

$$v_{g+1}^{i} = x_g^{NN_i} + F(x_g^{r_1} - x_g^{r_2}), \quad (1)$$

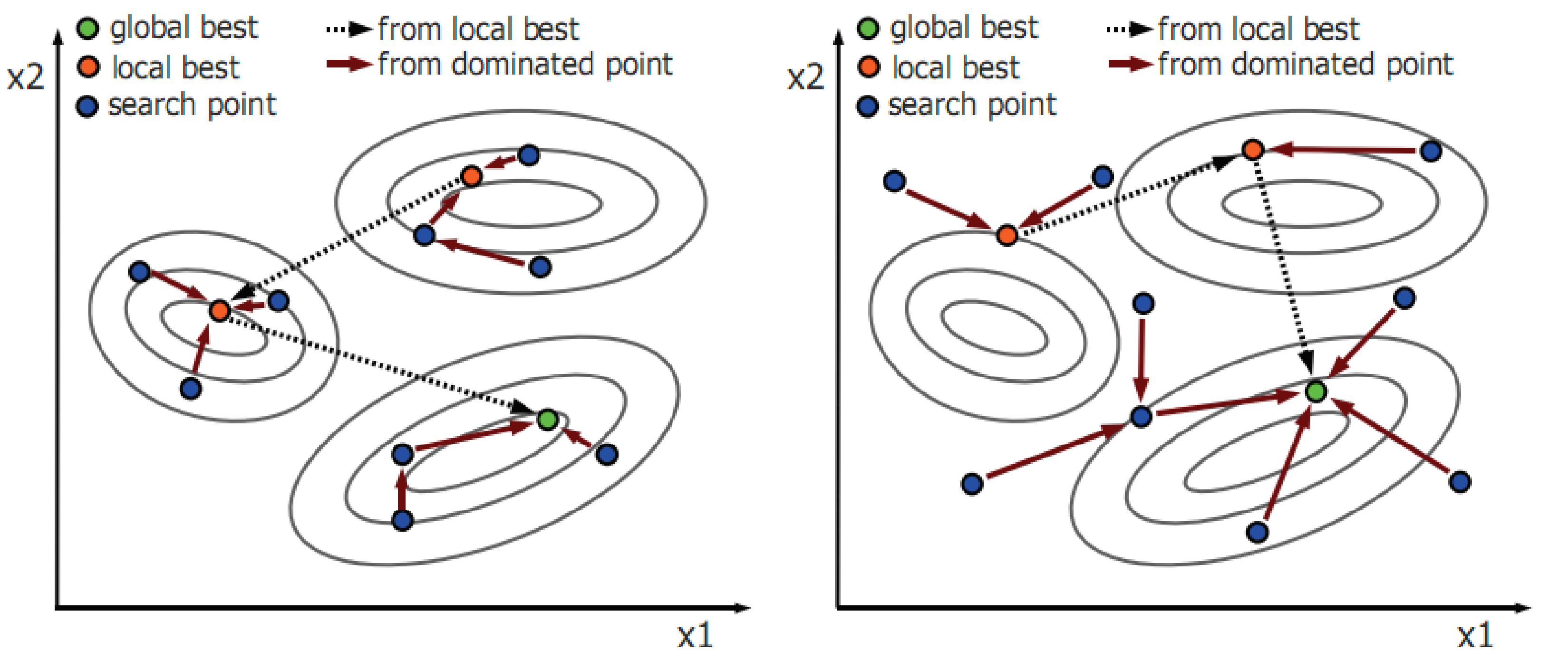
2) **"DE/nrand/2"**

$$v_{g+1}^{i} = x_g^{NN_i} + F(x_g^{r_1} - x_g^{r_2}) + F(x_g^{r_3} - x_g^{r_4}), \quad (2)$$

M. Epitropakis, D. Tasoulis, N. Pavlidis, V. Plagianakos, and M. Vrahatis, "Enhancing differential evolution utilizing proximity-based mutation operators," *IEEE Trans on Evol. Comput.*, vol. 15, no. 1, pp. 99–119, Feb 2011.

Nearest-better clustering

- **The basic idea**: basins of attraction are indicated by contour lines. Each individual connects to its nearest neighbour which is better; clustering is done via cutting the longest lines (Preuss 2010). However, still need to set a few niching parameters.
- This NBC (nearest-better clustering) idea combined with CMA-ES produces a niching algorithm that won the top place in the CEC’2013 niching competition.



M. Preuss. "Niching the CMA-ES via nearest-better clustering." In *Proceedings of the 12th annual conference companion on Genetic and evolutionary computation* (GECCO ’10). ACM, New York, NY, USA, pp. 1711-1718, 2010

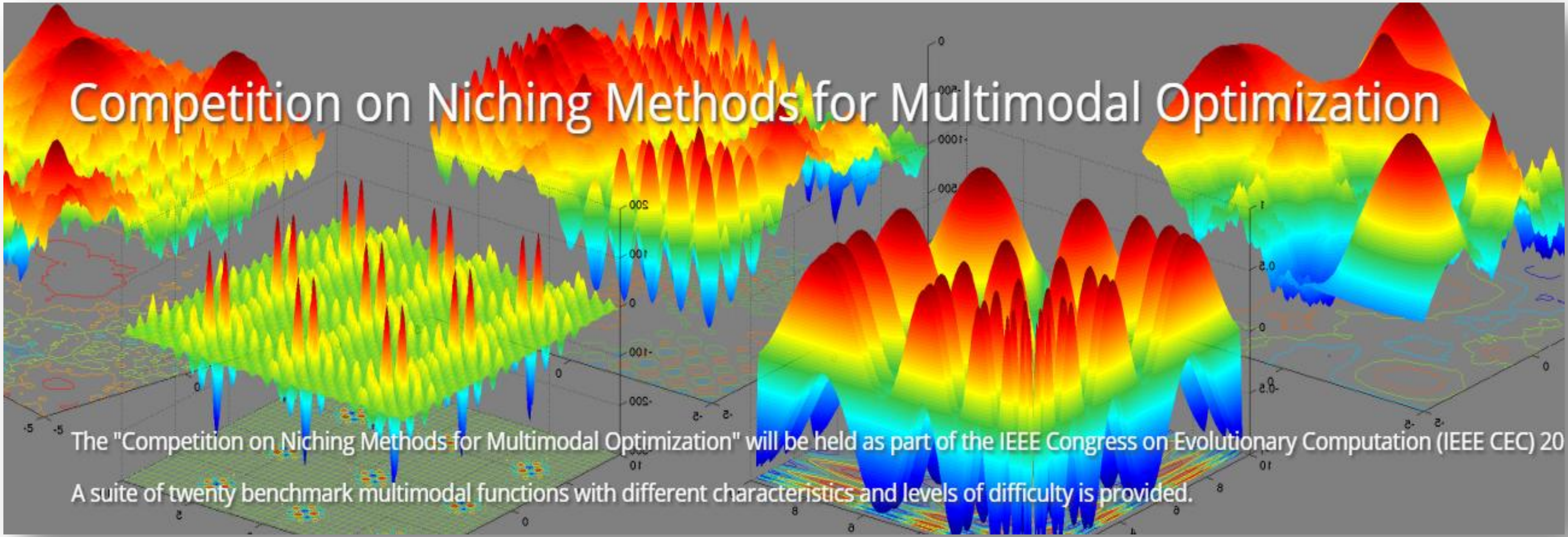
CEC'2013 niching benchmark

- A common platform for evaluating and comparing different niching algorithms.
- 20 benchmark multimodal functions with different characteristics.
- 5 accuracy levels: $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- The benchmark suite and the performance measures have been implemented in: **C/C++, Java, MATLAB**.

X. Li, A. Engelbrecht, and M.G. Epitropakis, "Benchmark Functions for CEC'2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization", Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013.

CEC 2013/2015/2016 competitions

- IEEE CEC niching competitions at 2013,2015 and 2016, with the latest results available at the following URL:
  - http://titan.csit.rmit.edu.au/~e46507/cec13-niching/competition/
  - http://titan.csit.rmit.edu.au/~e46507/cec15-niching/competition/
  - http://www.epitropakis.co.uk/cec16-niching/competition/
  - https://github.com/mikeagn/CEC2013



20 test functions

Id	Dim.	# GO	Name	Characteristics
F_1	1	2	Five-Uneven-Peak Trap	Simple, deceptive
F_2	1	5	Equal Maxima	Simple
F_3	1	1	Uneven Decreasing Maxima	Simple
F_4	2	4	Himmelblau	Simple, non-scalable, non-symmetric
F_5	2	2	Six-Hump Camel Back	Simple, not-scalable, non-symmetric
F_6	2,3	18,81	Shubert	Scalable, #optima increase with D, unevenly distributed grouped optima
F_7	2,3	36,216	Vincent	Scalable, #optima increase with D, unevenly distributed optima
F_8	2	12	Modified Rastrigin	Scalable, #optima independent from D, symmetric
F_9	2	6	Composition Function 1	Scalable, separable, non-symmetric
F_{10}	2	8	Composition Function 2	Scalable, separable, non-symmetric
F_{11}	2,3,5,10	6	Composition Function 3	Scalable, non-separable, non-symmetric
F_{12}	2,3,5,10	8	Composition Function 4	Scalable, non-separable, non-symmetric

Performance measures

Measures:

Peak Ratio (PR) measures the average percentage of all known global optima found over multiple runs:

$$\mathrm{PR} = \frac{\sum_{run=1}^{NR} \text{\# of Global Optima}_i}{(\text{\# of known Global Optima}) * (\text{\# of runs})}$$

Who is the winner:

- The participant with the highest average Peak Ratio performance on all benchmarks wins.
- In all functions the following holds: the higher the PR value, the better

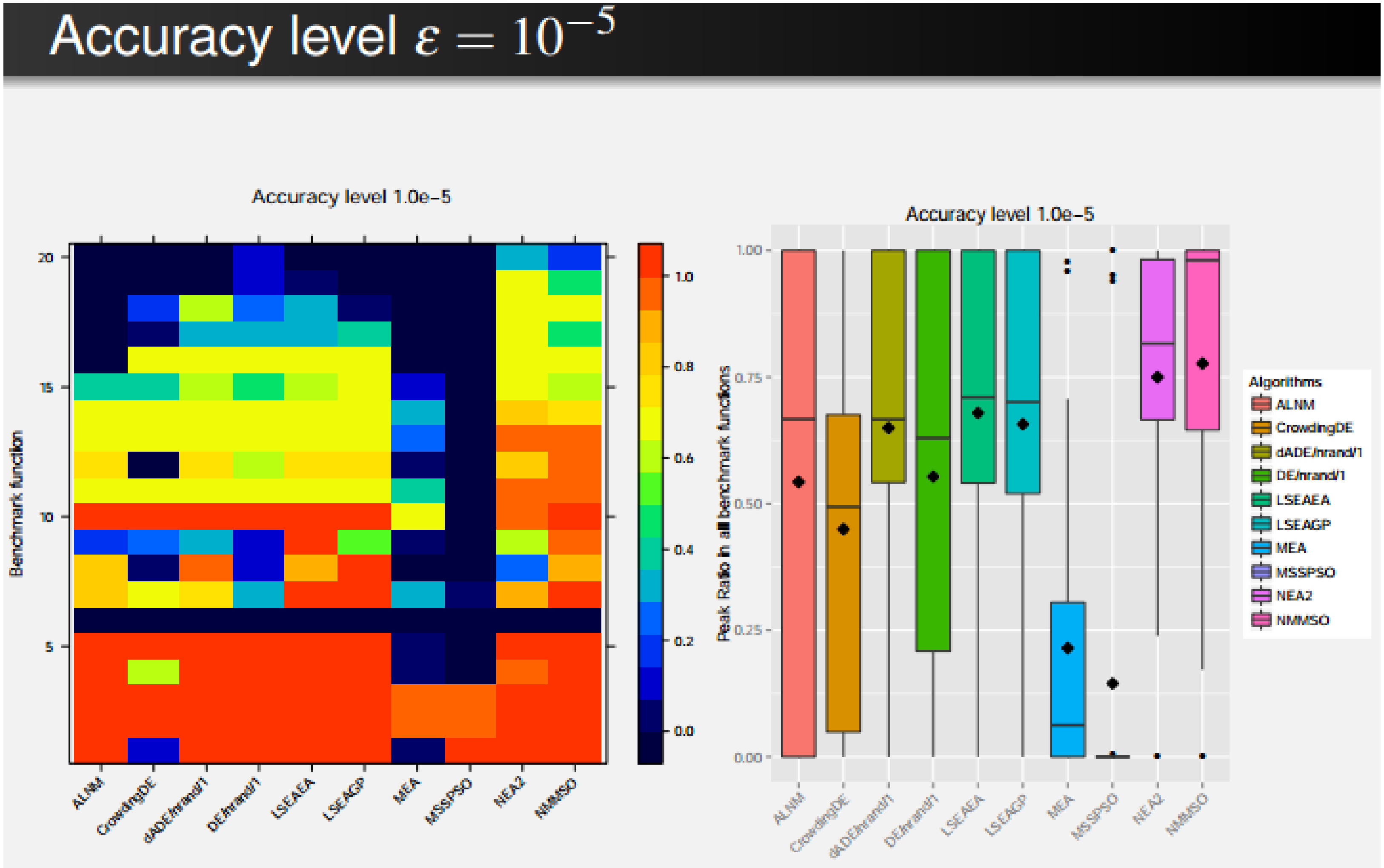
CEC 2013/2015 niching competition top 3 entries

- **(NMMO) Niching Migratory Multi-Swarm Optimiser:**
  - J. E. Fieldsend, "Running Up Those Hills: Multi-Modal Search with the Niching Migratory Multi-Swarm Optimiser," in IEEE Congress on Evolutionary Computation, 2014, pp. 2593 - 2600.

- **(NEA2) Niching the CMA-ES via Nearest-Better Clustering**:
  - M. Preuss. "Niching the CMA-ES via nearest-better clustering." In Proceedings of the 12th annual conference companion on Genetic and evolutionary computation (GECCO ’10). ACM, New York, NY, USA, pp. 1711-1718, 2010.

- **(LSEAGP) Localised Search Evolutionary Algorithm using a Gaussian Process**:
  - J. E. Fieldsend, "Multi-Modal Optimisation using a Localised Surrogates Assisted Evolutionary Algorithm," in UK Workshop on Computational Intelligence (UKCI 2013), 2013, pp. 88-95.

Algorithm performances



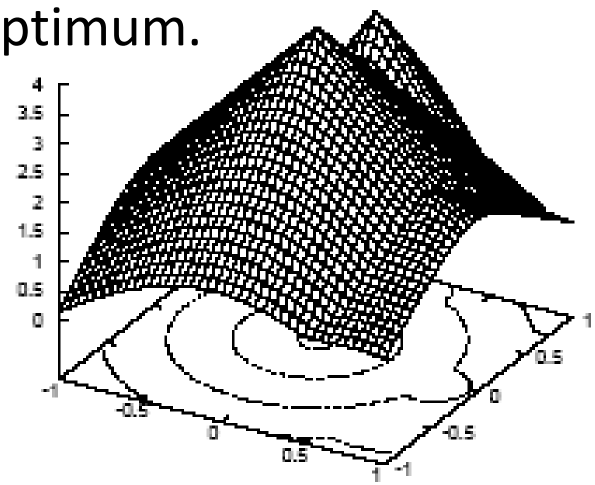
Participants' performance

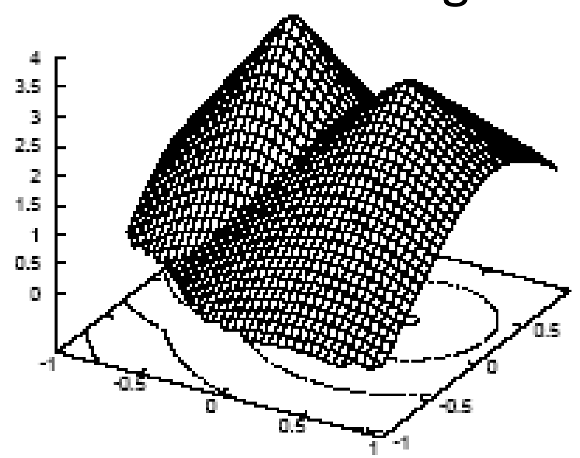
Algorithm	Statistics			Friedman's Test	
	Median	Mean	St.D.	Rank	Score
NMMSO	**0.9885**	**0.8221**	0.2538	**1**	16.1900
NEA2	**0.8513**	**0.7940**	0.2332	**2**	16.1150
LSEAEA	**0.9030**	**0.7477**	0.3236	**4**	14.5050
dADE/nrand/1	0.7488	**0.7383**	0.3010	5	14.2450
LSEAGP	**0.7900**	0.7302	0.3268	**3**	14.7550
CMA-ES	0.7550	0.7137	0.2807	6	14.0800
N-VMO	0.7140	0.6983	0.3307	7	13.7600
ALNM	0.7920	0.6594	0.3897	9	12.4900
PNA-NSGAII	0.6660	0.6141	0.3421	11	11.2700
NEA1	0.6496	0.6117	0.3280	14	10.5250
DE/nrand/2	0.6667	0.6082	0.3130	10	11.2950
dADE/nrand/2	0.7150	0.6931	0.3174	8	12.8100
DE/nrand/1	0.6396	0.5809	0.3338	13	10.6150
DELS-aj	0.6667	0.5760	0.3857	15	9.6950
CrowdingDE	0.6667	0.5731	0.3612	12	10.6200
DELG	0.6667	0.5706	0.3925	16	9.4400
DECG	0.6567	0.5516	0.3992	17	8.9900
IPOP-CMA-ES	0.2600	0.3625	0.3117	18	5.8700
MEA	0.2075	0.3585	0.3852	19	5.2750
A-NSGAII	0.0740	0.3275	0.4044	20	4.7200
MSSPSO	0.0000	0.2188	0.3913	21	3.7350

Niching in dynamic optimization

SPSO for tracking optima

- In contrast to optimization towards a static optimum, in a dynamic environment the goal is to **track** as closely as possible the **dynamically changing optima**.
- A useful strategy to ensure good tracking of the global optimum in a dynamic environment, is to maintain multiple species at all the optima found so far, regardless whether they are globally or locally optimal.
- By maintaining individual species at each local optimum, it helps tremendously in case when such a local optimum turns into a global optimum.





X. Li, J. Branke, and T. Blackwell, “Particle swarm with speciation and adaptation in a dynamic environment,” in Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, GECCO ’06. New York, NY, USA: ACM, 2006, pp. 51–58.

Niching in multiobjective optimization

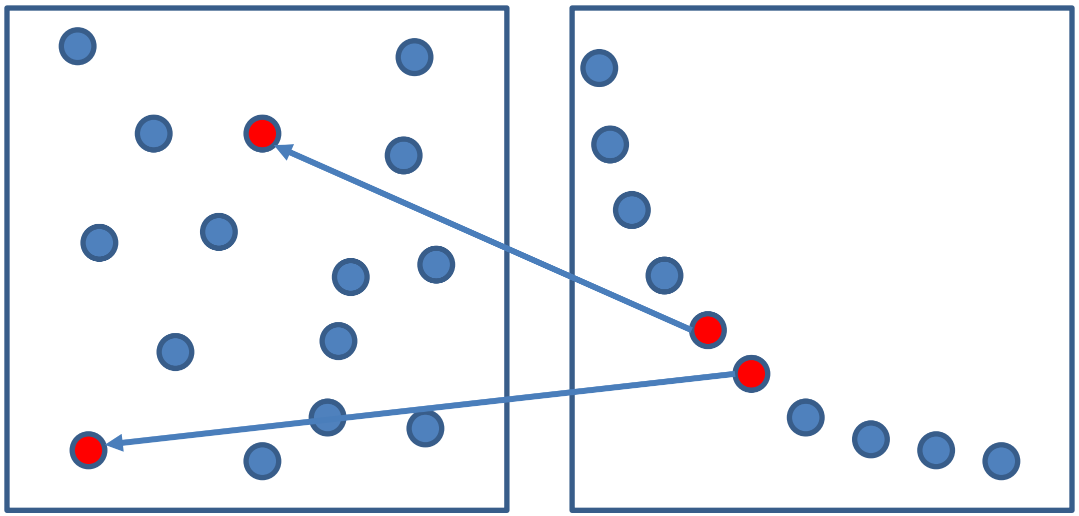
EMO solution diversity

- Although diversity maintenance is a much common issue in any population-based metaheuristics, it is possible to use niching methods for maintaining solution diversity. An early example is the Niched-Pareto GA (NGPA) (Horn, et al., 1994) , which is a multiobjective GA using a variant of fitness sharing to maintain Pareto solution diversity in the objective space. Another example is the crowding distance metric used in NSGA-II (Deb, et al., 2002).
- Much attention has been given to maintaining solution diversity in the objective space, however, little attention has been given to how to maintain solution diversity in the decision space. See next slide.

J. Horn, N. Nafpliotis, and D. E. Goldberg, "A Niched Pareto Genetic Algorithm for Multiobjective Optimization," in *Proc. of the First IEEE Conference on Evolutionary Computation,* vol. 1. IEEE Service Center, 1994, pp. 82–87.
K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: Nsga-ii," Evolutionary Computation, IEEE Transactions on, vol. 6, no. 2, pp. 182–197, Apr 2002.

Diversity in both spaces

- A multiobjective evolutionary algorithm (e.g., multiobjective Niching-CMA) can produce a much more diverse set of efficient solutions (i.e., solutions in the decision space), without sacrificing objective space diversity (Shir, et. al. 2009).



Decision space **Objective space**

An example where two solutions that are close in the objective space but their corresponding points in the decision space are further apart.

O. M. Shir, M. Preuss, B. Naujoks, and M. Emmerich, "Enhancing decision space diversity in evolutionary multiobjective algorithms," in Proceedings of the 5th International Conference on Evolutionary Multi-Criterion Optimization, ser. EMO '09. Berlin, Heidelberg: Springer- Verlag, 2009, pp. 95–109.

Omni-Optimizer

- Allows degeneration of NSGA-II into a single objective multimodal optimization method (i.e., a niching method).
- A variable space crowding distance metric is used to encourage distant solutions in the decision space to remain in the population.
- Distant solutions with similar or equal objective function values will survive.
- **Omni-Optimizer** can degenerate to a niching method for multiobjective multimodal optimization, capable of finding multiple Pareto-optimal fronts.

K. Deb and S. Tiwari, “Omni-optimizer: A generic evolutionary algorithm for single and multi-objective optimization.” European Journal of Operational Research, vol. 185, no. 3, pp. 1062–1087, 2008.

IEEE CIS Taskforce on MMO

- **The key objective** is to promote research on multi-modal optimization, including its development, education and understanding of sub topic areas of multi-modal optimization. Further info: http://www.epitropakis.co.uk/ieee-mmo/
- **Current chair**: Michael G. Epitropakis (Lancaster University, UK).
- **Vice-Chairs**: Andries Engelbrecht (University of Pretoria, South Africa), and Xiaodong Li (RMIT University, Australia).
- **Members**: Carlos A. Coello Coello, Kalyanmoy Deb, Andries Engelbrecht, Michael G. Epitropakis, Jonathan Fieldsend, Jian-Ping Li, Xiaodong Li, Jonathan Mwaura, Konstantinos Parsopoulos, Vassilis Plagianakos, Mike Preuss, Bruno Sareni, Ofer M. Shir, Patrick Siarry, P. N. Suganthan, Michael N. Vrahatis, Bo-Yang Qu, Simon Wessing, Xin Yao.
- **Past and planned activities:**
  - IEEE CEC 2010, 2013, and 2015 special session and/or competitions on “Niching Methods for Multimodal Optimization”.
  - International Workshop on "Advances in Multimodal Optimization", PPSN 2014.
  - Tutorial at WCCI 2016.
  - A repository for publications and source codes.

Summary

- Niching methods have been studied for the past few decades, and now experience a revival, as more people from diverse backgrounds find its relevance in their own disciplinary areas.
- Niching methods can be developed using other meta-heuristics, apart from evolutionary algorithms.
- Niching has its application in many problem solving domains, e.g., dynamic optimization and multiobjective optimization.
- A good starting point for new comers: several survey papers are available, plus recently a new book by Mike Preuss.
- Many open research questions and challenges to be addressed.
- Many possible real-world applications of niching methods.

S. Das, S. Maity, B.-Y. Qu, and P. Suganthan, “Real-parameter evolutionary multimodal optimization - a survey of the state-of-the-art,” *Swarm and Evolutionary Computation*, vol. 1, pp. 71–88, June 2011.
O. Shir, “Niching in evolutionary algorithms,” *Handbook of Natural Computing: Theory, Experiments, and Applications*, pp. 1035–1069, 2012.
Li, X., Epitropakis, M.G., Deb, K., and Engelbrecht, A. (2017), "Seeking Multiple Solutions: an Updated Survey on Niching Methods and Their Applications", *IEEE Transactions on Evolutionary Computation* (accepted, 01/12/2016).

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