A two-stage stochastic programming model for inventory management in the blood supply chain

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Introduction

Background

Managing inventories in the blood supply chain (BSC) is a challenging task due to:

- uncertain supply and demand
- perishible nature of blood products
- eight blood types (O+, O-, A+, A-, B+, B-, AB+, AB-)
- cross-matching
- shortages may result in loss of life
- strong subjective bias against shortage/outdating, not just cost minimisation

The BSC can be looked at as a whole, from donation to transfusion, or as a subsystem.

It can be loosely split into the following chain [Osorio et al., 2015], however note that in reality the system is not linear, and is in fact highly complex

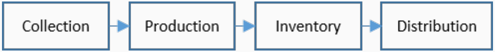


Figure 1: Main echelons of the blood supply chain

Problem description

The underlying problem consists of defining an ordering rule to be followed by the hospital's inventory management team.

When uncertainty must be taken into consideration in inventory management decisions, several inventory control policies can be found in the literature, Axsäter [2015]

Classically, such policies can be classified as (s, Q), (s, S), (R, Q), (R, S), and (R, s, S)

In systems that use control policy (R, S), in every R time units (review periodicity), a variable quantity sufficient to raise the stock level to the S position is ordered

In our model the inventory reference level to be used as a target for each blood type is $S = (S_\beta)_{\beta \in B}$, where $\beta \in B$ represents the collection of distinct blood types

Two main obstacles arise when considering the definition of an optimal (R,S) policy for RBC inventories, namely the nature of the stochastic process driving the demand and the inherent perishable nature

To address the problem, we propose an optimisation model based on the multi-period multi-product lot-sizing problem under demand uncertainty that is subject to operate under a (R,S)-based replenishment policy and considers product perishability

Whose objective is to minimise costs considering constraints on inventory balance and perishability

Two-stage stochastic programming model

We propose a 2SSP model with recourse, that takes into account perishability and demand uncertainty, for defining optimal periodic review policies (R, S) for red blood cells (RBCs).

The model focuses on minimising operational costs, as well as blood shortages and wastage due to outdating.

In a 2SSP model the decision variables are grouped into first- and second-stage decisions. The first-stage must be made before we know the demand, and the second-stage are made after the uncertainty is revealed.

The aim is to make first-stage decisions that must hold for all scenarios considered.

Model formulation

We first constructed the natural mixed-integer nonlinear programming (MINLP) version of the model.

$$\min \quad \sum_{\rho} O_\rho v_\rho + \sum_{\xi} P(\xi) \left[\sum_{\rho,\beta} \left(H_\rho i(\xi)_{\rho,\beta} + T_\rho f(\xi)_{\rho,\beta} + K_\rho e(\xi)_{\rho,\beta} + \sum_{\beta'} Q_{\beta',\beta} p(\xi)_{\rho,\beta',\beta} \right) \right] \tag{1}$$

Inventory decision constraints

$$\text{s.t.: } \sum_{\gamma} u_{\gamma} = 1 \tag{2}$$

$$\sum_{\gamma} W_{\rho,\gamma} u_{\gamma} = v_{\rho}, \qquad \forall \rho \tag{3}$$

$$0 \leq s_{\beta} \leq \overline{S}_{\beta}, \qquad \forall \beta \tag{4}$$

Demand fulfillment constraints

$$q(\xi)_{\rho,\beta} = (s_\beta - it(\xi)_{\rho-1,\beta})v_\rho, \qquad \forall \rho \leq |P| - (L+1), \forall \beta, \forall \xi \tag{5}$$

$$q(\xi)_{\chi-L,\beta} = \sum_{\rho=\chi}^{\theta_\chi} c(\xi)_{\chi,\rho,\beta} + e(\xi)_{\theta_\chi,\beta}, \qquad \forall \chi > L, \forall \beta, \forall \xi \tag{6}$$

$$\sum_{\chi=\pi_\chi}^{\rho} c(\xi)_{\chi,\rho,\beta} = \sum_{\beta'} C_{\beta',\beta} p(\xi)_{\rho,\beta',\beta}, \qquad \forall \rho, \forall \beta, \forall \xi \tag{7}$$

$$\sum_{\beta'} p(\xi)_{\rho,\beta',\beta} + f(\xi)_{\rho,\beta} = D(\xi)_{\rho,\beta}, \qquad \forall \rho, \forall \beta, \forall \xi \tag{8}$$

θ_ρ - Last period in which blood units (received at period ρ) can be used, i.e. $\theta_\rho = \min\{\rho + M - 1, |P|\}$
π_ρ - Earliest period in which blood units can be acquired and still be used in period ρ, i.e. $\pi_\rho = \max\{1, \rho - M + 1\}$

Inventory balance constraints

$$i(\xi)_{\rho,\beta} = \sum_{\chi=\pi_{\rho+1}}^{\rho} \sum_{\tau=\rho+1}^{\theta_\chi} c(\xi)_{\chi,\tau,\beta}, \qquad \forall \rho \leq |P| - 1, \forall \beta, \forall \xi \tag{9}$$

$$it(\xi)_{\rho,\beta} = i(\xi)_{\rho,\beta} + \sum_{\chi=\rho-L+1}^{\rho} q(\xi)_{\chi,\beta}, \qquad \forall \rho \leq |P| - 1, \forall \beta, \forall \xi \tag{10}$$

Domain of variables

$$u_\gamma \in \{0,1\}, \qquad \forall \gamma \tag{11}$$

$$v_\rho \in \{0,1\}, \qquad \forall \rho \tag{12}$$

$$e(\xi)_{\rho,\beta}, f(\xi)_{\rho,\beta}, i(\xi)_{\rho,\beta}, it(\xi)_{\rho,\beta}, q(\xi)_{\rho,\beta} \in \mathbb{Z}^+, \qquad \forall \rho, \forall \beta, \forall \xi \tag{13}$$

$$p(\xi)_{\rho,\beta',\beta} \geq 0, \qquad \forall \rho, \forall \beta', \forall \beta, \forall \xi \tag{14}$$

Linearisation

Due to the inherent complex nature of this (MINLP) class of problem we reformulated the nonlinear constraint (5) to obtain an exact linearised version (15-17), due to the binary nature of v_ρ

Here, exact means that it presents the same global optimal solution as its nonlinear counterpart.

$$q(\xi)_{\rho,\beta} = (s_\beta - it(\xi)_{\rho-1,\beta})v_\rho$$

$$q(\xi)_{\rho,\beta} - (s_\beta - it(\xi)_{\rho-1,\beta}) \leq \overline{S}_\beta(1 - v_\rho) \tag{15}$$

$$q(\xi)_{\rho,\beta} - (s_\beta - it(\xi)_{\rho-1,\beta}) \geq \overline{S}_\beta(v_\rho - 1) \tag{16}$$

$$q(\xi)_{\rho,\beta} \leq \overline{S}_\beta v_\rho \tag{17}$$

Case Study

Case Description

The proposed model was validated using a case study of a medium-sized hospital that is responsible for defining its inventory target level S and replenishment frequency R.

To simulate the use of demand forecasting by the hospital, we considered a planning horizon of 93 days (12-weeks).

The hospital currently places an order every day ($R = 1$) and adopts a target level ($S = 173$) of eight times the average daily demand, with a lead time of $L = 1$.

Demand generation

The actual demand forecast could not be made available, so realistic data representing the daily demand was generated using informed average demand and its standard deviation, and the assumption that uncertain demand follows a negative binomial distribution [Nahmias, 2011]

Once the statistical distribution that models daily demand was defined, scenario sets were created using Monte Carlo sampling. Figure 2 presents 10 demand scenarios for O+.

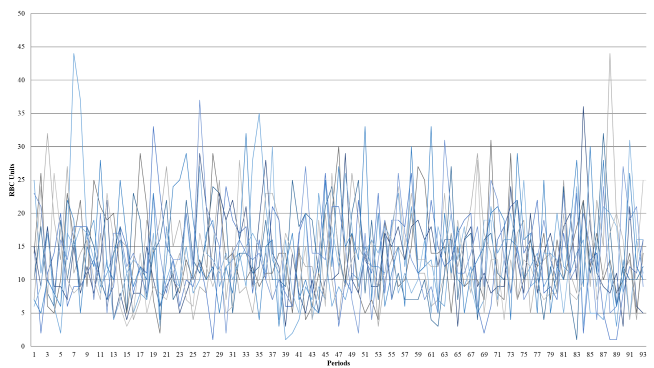


Figure 2: 10 samples of O+ demand scenarios

Figure 3: Out-of-sample

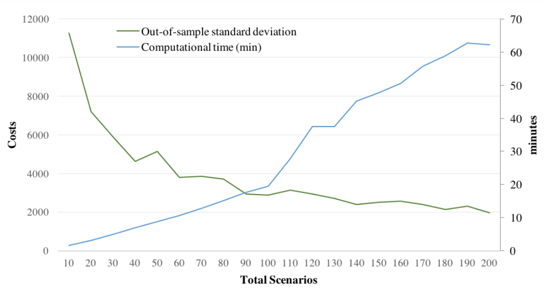
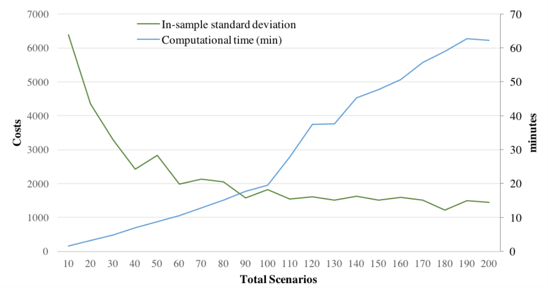


Figure 4: In-sample



Numerical experiments

We performed three types of experiments:

- single blood multi-objective
- single blood cost-based
- multi-blood

Sensitivity analysis was performed for different holding/ordering cost ratios, service levels, and outdate limits.

A typical instance of the proposed model had 83,250 constraints and 55,850 variables (103 of which being integer).

Results

Single blood multi-objective experiments

The model was used to obtain insights from assessing the implications of performing a cost-oriented management approach (Cost-then-Outdate, CtO) or a minimal-outdate-oriented management approach (Outdate-then-Cost, OtC)

Note: Total cost is the percentage of the hospital's current policy (100%) performance

Cost ratio	Service level	Outdate limit	CtO			OtC		
			R	S	Total cost	R	S	Total cost
0.005	95%	10%	5	102	25%	1	43	64%
	99%	5%	5	128	32%	1	64	70%
0.01	95%	10%	4	87	27%	1	43	49%
	99%	5%	4	94	36%	1	64	58%
0.02	95%	10%	3	71	26%	1	43	36%
	99%	5%	3	94	39%	1	64	46%

Table 1: Single blood multi-objective results

The current inventory policy for the case study hospital sets R= 1 and S = 173

For the CtO experiments:

- the frequency of reviews (R) is increased in all cases
- in all cases S is reduced to a maximum of S=128 for cost ratio 0.005 with a service level of 99%, and a minimum of S=71 for cost ratio 0.02 with a service level of 95% for the optimised policy
- the outdate limit did not affect the defined policies.

For the OtC experiments:

- the model becomes insensitive to the cost ratios defined and decides for R=1 in all cases
- the optimal S levels are 43 for service levels of 95% and 64 for 99%
- the outdate limit did not affect the defined policies.

Key findings:

- the CtO can reduce the total cost of the current policy by 61%-75%, while increasing the review periods to 3-5 days, and reducing the target inventory level
- the OtC can reduce the total cost by 30%-64%, while the review periods stay the same at 1 day, and the target inventory level decreases by over half.

Single blood cost-based experiments

Unlike the previous experiments performed with cost ratios to represent the cost parameters, in these cost-based experiments we ran the model once, without enforcing hierarchy between cost and outdate minimisation objectives or hard limits on the service level or outdate.

We also modified the ordering cost (originally $\sum_{\rho} O_{\rho} v_{\rho}$) to be $\sum_{\rho,\xi} P(\xi) O_{\rho} q(\xi)_{\rho}$, so it considers the ordering cost per unit instead of per order.

$O_{\rho} = \$180/unit$, $T_{\rho} = \$1500/unit$, $K_{\rho} = \$150/unit$, $H_{\rho} = \$1.25/unit/day$

In this set of experiments we compared three distinct policies:

- the current policy representing R=1 and S=173,
- an optimised policy in which the model decides for optimal values for R and S,
- a semi-optimal policy in which we set R=1, however let the model define a optimal value for S.

	Optimised policy	Semi-optimal policy	Current policy
R	7	1	1
S	152	59	173
Total cost	$221,281	$228,693	$342,010
Avg. age (days)	5.95	2.27	9.92
Avg. outdate	0	0	0
Avg. shortage	0.020	0.016	0.000
Avg. inventory	80.60	30.70	134.60

Table 2: Single blood cost-based results

Key findings:

- a difference of over $100,000 between the cost associated with the current policy and the semi- and optimised policies
- total unmet demand for current policy is zero, while slightly more for optimised policies
- the inventory levels smaller in optimised policies, which suggests this may be a main factor causing increased costs

Multiple blood experiments

These results consider the eight ABO and Rh blood groups simultaneously, and only concentrate on minimising costs, with R=1 acting as a proxy for minimising outdate

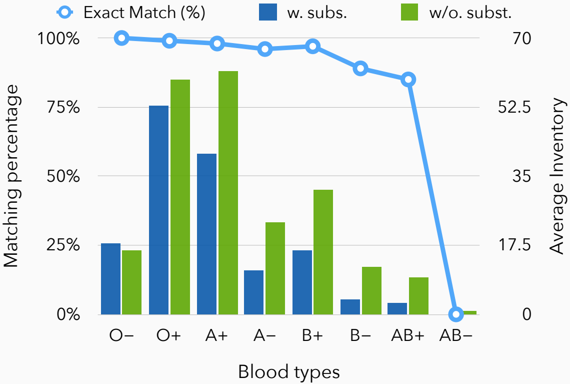
We consider cases where substitution is and is not allowed. Where substitution is allowed, the following priorities hold stated in .

Blood Type	Demand Proportion (%)	Priority Choice 1st	2nd	3rd	4th	5th	6th
O+	39.55	O-					
O-	9.38	None					
A+	31.28	A-	O+	O-			
A-	5.59	O-					
B+	11.20	B-	O+	O-			
B-	1.35	O-					
AB+	1.51	A+	B+	A-	B-	O+	O-
AB-	0.10	A-	B-	O-			

Table 3: Blood substitution priorities [Abbasi et al., 2017]

	w/o substitution		with substitution	
	S	Avg. age	S	Avg. age
O+	91	4.16	84	3.66
O-	24	4.80	26	4.90
A+	89	5.43	65	3.62
A-	30	11.46	16	5.21
B+	43	7.79	25	4.11
B-	20	24.22	5	7.43
AB+	13	17.11	4	6.04
AB-	5	25.98	0	-

- the target levels and average inventory for with substitution are reduced
- the average age of the RBCs on issue is also considerably reduced as a results of a more efficient use of the blood units.



- exact match is acceptable for all blood types apart from AB- (S=0), this situation is not realistic as it is a priority to supply patients with their own blood type.

Conclusion

Summary

- proposed a two-stage stochastic programming model
- takes into account ordering, holding, outdate, and shortage costs to define optimal (R,S) policy
- considers demand uncertainty, represented by a set of discrete scenarios
- considers perishability of blood
- fairly flexible and could be easily extended to different planning horizons, various lead times, and shorter or longer shelf lives that are applicable to other products

Main insights

The model could be used as a decision support tool for the management of blood inventory

- possible to modify current policy by reducing S without compromising the service provided
- at the same time minimising outdate, age of issue, and holding costs
- considering multiple blood types with substitution further improves the performance - with the total cost of the model roughly 40% lower

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Questions?