Optimising the Resilience of Road Networks under Uncertainty

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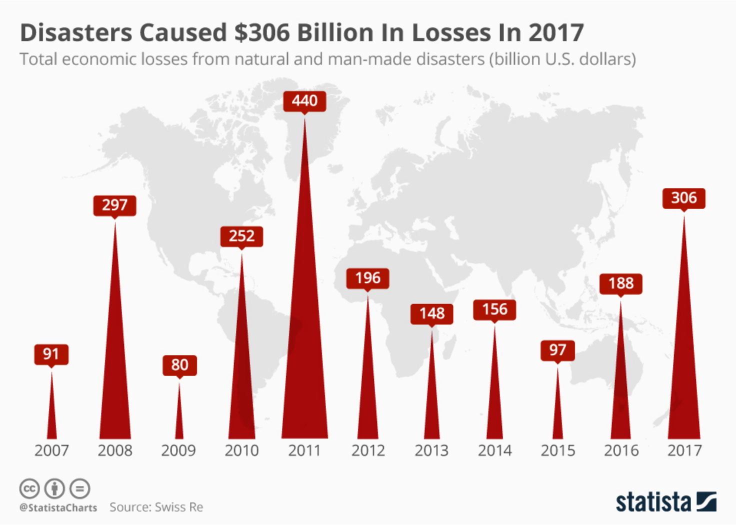
Outline

Outline

Disasters
Economic damages



Roads under disasters

Roads under disasters
Queensland wet seasons

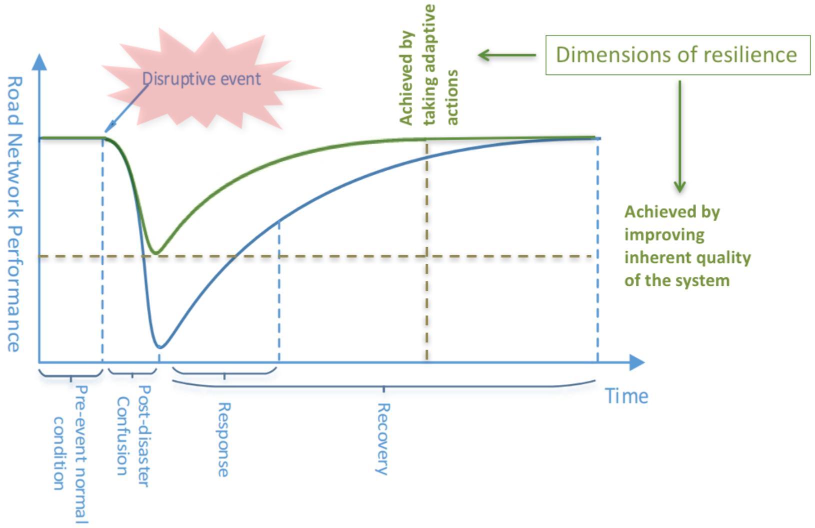
- From 2009-10 to 2011-12, road network sustained $9 billion in damage in Queensland.
- Of which $5 billion funded through Natural Disaster Relief.

Disaster Management

Life-cycle

Source: Erskine & Gregg, 2012

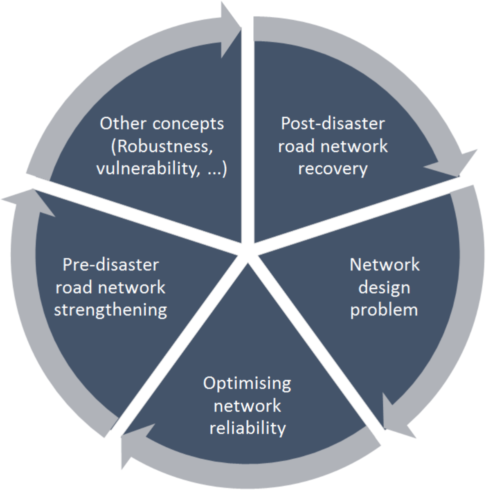
Roads under disasters
Dimensions of resilience



Outline

Literature

Literature topics on optimising road network resilience



Literature

A summary of the resilience-related terminologies for road networks

Terminology	Definition
Resilience	Inherent ability of the road network to resist against a disaster as well as adapting actions taken during post-event to reduce the impacts.
Reliability	The probability that a road network is capable of meeting its pre-defined expectations and goals.
Vulnerability	The susceptibility of road network to disruptive events of any kind that can reduce the serviceability of it.
Risk	The probability of destruction and damages in a road network imposed by a hazard over a period of time.
Robustness	The amount that a road network can retain its expected functionality.
Survivability	The extent to which a road network can continue its mission under given damages to its compartments.
Flexibility	The strength of a road network to maintain its satisfactory performance under external changes
Criticality	The extent to which the community and businesses are dependant on a road network or some of its elements.
Serviceability	The likelihood to use a road network over a period of time.

NDP

- It deals with making optimal decisions about the expansion of road network infrastructure (Yang & H. Bell, 1998).
- CNDP, DNDP and MNDP

NDP under uncertainty

- Scenario-based (Multi-stage SP) (Wang & Xie, 2016)
- Robust (minimax, Uncertainty sets) (Yin, Madanat, & Lu, 2009)
- Chance-constrained (Chen & Xu, 2012)
- Probability models (+ alpha-reliable models and Meanvariance models) (Unnikrishnan & Lin, 2012; Yin & Ieda, 2002)

Road network strengthening

Pre-event optimisation problems aime at finding the best strengthening activities for the network (Asadabadi & Miller-Hooks, 2017; Faturechi & Miller-Hooks, 2014a; Fan & Liu, 2010).

NDP as a means for optimising network strengthening

Usually formulated as a bi-level NDP.

Outline

Problem formulation
Resilience indicators

Demand satisfaction resilience

$$R_k^D = 1 - \frac{\phi_k^2(\xi_2)}{\sum_{w \in W} \sigma_{k,w}(\xi_1) q_w(\xi_1)}, \forall k \in K \qquad (1)$$

Travel time resilience

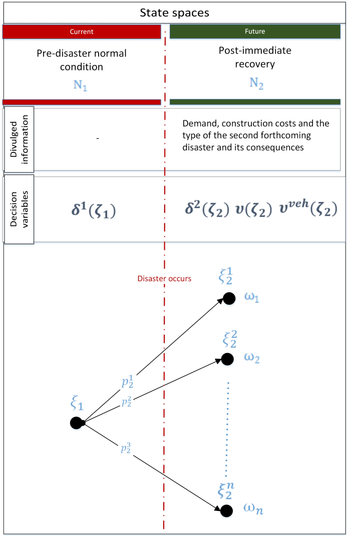
$$R_k^{TT} = \frac{TT_k^1(\xi_1)}{TT_k^2(\xi_2)}, \forall k \in K \qquad (2)$$

$$TT_k^2(\xi_2) = \sum_{a \in A} \upsilon_{a,k}(\xi_2) t_a(\upsilon_a(\xi_2), c_a(\xi_2)), \forall k \in K \qquad (3)$$

$$t_a(v_a, c_a) = t_a^0 . \left(1 + 0.15 \left(\frac{v_a}{c_a}\right)^4\right) \qquad (4)$$

Problem formulation

Scenario tree of two-stage Stochastic Mathematical Program



Problem formulation
Upper-level

$$\max_{\delta^1(\xi_1)} \Big[\mathbf{E}\Big[\max_{\delta^2(\xi_2),\upsilon_a(\xi_2)} R_1^{TT} \Big], \ldots, \mathbf{E}\Big[\max_{\delta^2(\xi_2),\upsilon_a(\xi_2)} R_K^{TT} \Big], \mathbf{E}\Big[\max_{\delta^2(\xi_2),\upsilon_a(\xi_2)} R_1^{D} \Big], \ldots, \mathbf{E}\Big[\max_{\delta^2(\xi_2),\upsilon_a(\xi_2)} R_k^{D} \Big] \Big] \tag{5}$$

s.t.

$$c_a(\xi_2) = c_a(\xi_1) + \delta_a^{1,e}(\xi_1) + \Big[\sum_{z \in Z} \Big(\varkappa_z(\xi_2) \Big(cl_{a,z}(\xi_2)(c_a(\xi_1) + \delta_a^{1,e}(\xi_1))(\delta_{a,z}^{1,ret}(\xi_1) - 1) \Big) + \delta_{a,z}^2(\xi_2) \Big) \Big], \forall a \in A \tag{6}$$

$$\sum_{a \in A} \delta_a^{1,e}(\xi_1) b_a^e(\xi_1) + \sum_{z \in Z} \varkappa_z(\xi_2) \sum_{a \in A} \delta_{a,z}^{1,ret}(\xi_1) b_{a,z}^{ret}(\xi_1) + \sum_{z \in Z} \varkappa_z(\xi_2) \sum_{a \in A} \delta_{a,z}^2(\xi_2) b_{a,z}^{rec}(\xi_2) \leq B \tag{7}$$

$$0 \leq c_a(\xi_2) \leq c_a(\xi_1), \forall a \in A \tag{8}$$

$$\delta_{a,z}^2(\xi_2) t_{a,z}^{rec} \leq T^{rec}, \forall a \in A, \forall z \in Z \tag{9}$$

$$\sum_{a \in A} \sum_{z \in Z} \delta_{a,z}^2(\xi_2) p_{a,z}^{rec} \leq P^{rec} \tag{10}$$

$$0 \leq \delta_a^{1,ret}(\xi_1) \leq 1, \forall a \in A \tag{11}$$

$$\upsilon_a(\xi_2) \geq 0, \forall a \in A \tag{12}$$

Problem formulation

Lower-level: UE equilibrium

A variational inequality problem (Wu, Florian, & He, 2006; Kaviani, Thompson, Rajabifard, & Sarvi, 2018):

$$\sum_{w \in W} \sum_{k \in K} \sum_{r \in R_W} [\rho^{w}_{r,k}(\xi_2)](f^*)(f_r - f_r^*) \geq 0, \forall f \in \Lambda \quad (13)$$

Λ is the feasible set defined by:

$$\sum_{r \in R_W} f^{w}_{r,k}(\xi_2) = \sigma_{k,w}(\xi_1) q^{'}_{w}(\xi_2), \forall w \in W, \forall k \in K \quad (14)$$

$$\upsilon_{a,k}(\xi_2) = \sum_{w \in W} \sum_{r \in R_W} f^{w}_{r,k}(\xi_2) \kappa_{k,a,r,w}, \forall a \in A, \forall k \in K \quad (15)$$

$$\upsilon_a(\xi_2) = \sum_{w \in W} \sum_{r \in R_W} \sum_{k \in K} PCE^k . \upsilon_{a,k}(\xi_2), \forall a \in A \quad (16)$$

$$f^{w}_{r,k}(\xi_2) \geq 0, \forall r \in R_W, \forall w \in W, \forall k \in K \quad (17)$$

$$\sigma_{k,w}(\xi_1) q_w(\xi_2) \geq \sigma_{k,w}(\xi_1) q_w(\xi_1), \forall w \in W, \forall k \in K \quad (18)$$

$$\rho^{w}_{r,k}(\xi_2) = \sum_{a \in A} t_a(\upsilon_a(\xi_2), c_a(\xi_2)) \kappa_{k,a,r,w}, \forall k \in K, \forall r \in R_W, \forall w \in W \quad (19)$$

$$\phi^2_k(\xi_2) = \sum_{w \in W} (\sigma_{k,w}(\xi_1) q_w(\xi_2) - \sum_{r \in R_W} f^{w}_{r,k}(\xi_2)), \forall k \in K \quad (20)$$

Outline

Conversion to single-level, mono-objectives

WSM

$$\max_{\delta^1(\xi_1)} \left[\mathbf{E} \left[\max_{\delta^2(\xi_2), \upsilon_a(\xi_2)} \sum_{k \in K} M w_k^R R_k^D + w_k^R R_k^{TT} \right] \right] \tag{21}$$

s.t.

$$\sum_{k \in K} w_k^R = 1 \tag{22}$$

+ the rest of constraints

DUE constraints

Reduced into a single-level one by adding the Karush-Kuhn-Tucker (KKT) conditions into the upper-level (Wang & Lo, 2010).

$$\begin{cases} f_{r,k}^w(\xi_2)\big(\rho_{r,k}^w(\xi_2) - \mu_k^w(\xi_2)\big) = 0 \\ \rho_{r,k}^w(\xi_2) - \mu_k^w(\xi_2) \geq 0 \end{cases} \quad \forall k \in K, \forall r \in R_w, \forall w \in W \tag{23}$$

Reformulation of the DUE constraints

- Disjunctive constraints (Wang & Lo, 2010)
- To circumvent the problems of disjunctive constraints (i.e. a large constant), (Siddiqui & Gabriel, 2013) propose **Schur's decomposition** and Special Ordered Set (SOS) type 1 variables for solving the MPEC.

$$\begin{cases} \boldsymbol{u}(\xi_2) = \frac{\boldsymbol{f}(\xi_2)+(\boldsymbol{\rho}(\xi_2)-\boldsymbol{\mu}(\xi_2))}{2} \\ (\boldsymbol{v}(\xi_2)^+ - \boldsymbol{v}(\xi_2)^-) = \frac{\boldsymbol{f}(\xi_2)-(\boldsymbol{\rho}(\xi_2)-\boldsymbol{\mu}(\xi_2))}{2} \\ \boldsymbol{u}(\xi_2) - (\boldsymbol{v}(\xi_2)^+ + \boldsymbol{v}(\xi_2)^-) = 0 \end{cases} \tag{24}$$

- ($\boldsymbol{f}(\boldsymbol{\xi_2})$,$\boldsymbol{\rho}(\boldsymbol{\xi_2})$ and $\boldsymbol{\mu}(\boldsymbol{\xi_2})$) are vectors of flows, path travel times and shortest travel times
- Vector $\boldsymbol{u}(\xi_2)$ and $\boldsymbol{v}(\xi_2)$ are defined.
- $\boldsymbol{v}(\xi_2)^+$ and $\boldsymbol{v}(\xi_2)^-$ are SOS type 1 variables

Linearisation of the polynomial link performance function

$$t_a(\upsilon_a(\xi_2), c_a(\xi_2)) = t^0.\left(1 + 0.15\left(\frac{\upsilon_a(\xi_2)}{c_a(\xi_2)}\right)^4\right) \qquad (25)$$

Techniques

- First-order Taylor series. (Wang & Lo, 2010)
- Transforming into logarithmic functions for which an outer-approximation technique is required to solve the non-linear problem. (Liu & Wang, 2015)
- Logarithmic convex combination (Log). The best in simplex-based approximations. (Faturechi & Miller-Hooks, 2014b)
- SOS2 (Special Ordered Set type 2). The best hypercube-based approximations according to (Silva & Camponogara, 2014).

Linearisation of the polynomial link performance function

Rectangle activation

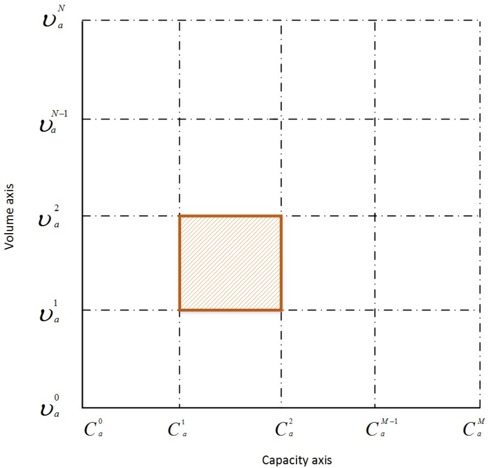


Figure: Single active square in a 2D domain space with capacity and volume as dimensions.

Linearisation of the polynomial link performance function

Simplex activation

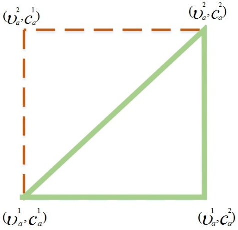


Figure: Activated simplex.

Linearisation of the polynomial link performance function
Interpolation

Last step is interpolation of the vertices of the activated simplex.

$$\begin{cases} \hat{t}_a(\upsilon_a(\xi_2), c_a(\xi_2)) = \sum_{i=0}^{M} \sum_{j=0}^{N} \theta_a^{i,j}.t_a(v_a^j, c_a^i), \\ v_a = \sum_{i=0}^{M} \sum_{j=0}^{N} \theta_a^{i,j}.v_a^j, \\ C_a = \sum_{i=0}^{M} \sum_{j=0}^{N} \theta_a^{i,j}.C_a^i, \\ \sum_{i=0}^{M} \sum_{j=0}^{N} \theta_a^{i,j} = 1, \forall i \in 0, 1, ..., M, j \in 0, 1, ..., N, \forall a \in A \\ \theta_a^{i,j} \geq 0, \end{cases} \tag{26}$$

Linearisation of the polynomial link performance function
Log model

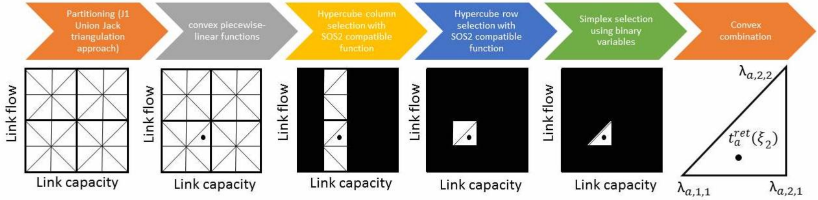


Figure: Steps in Log model.

This second method outperforms!

Linear approximation of bi-linear terms

Methods

- Incremental cost (IC)
- Convex combination (CC)
- SOS-based (Type 1 and 2)
- Uni-variate or bi-variate partitioning schemes

We used bi-variate SOS type 1 that has the best performance according to (Hasan & Karimi, 2010).

Linear relaxation of the objective function

- We employ the method used by (Wang & Lo, 2010) and (Faturechi & Miller-Hooks, 2014b) for linearisation of the objective function. In this method, the objective function is linearised through the use of shortest travel times ($\mu_k^w(\xi_2)$) and the demand ($\sigma_{k,w}(\xi_1)q_w(\xi_2)$).

Outline

Stochastic programming methods

- L-shaped decomposition (Van Slyke & Wets, 1969) - a cutting-plane method that can be seen as a special case of generalised Bender's decomposition method.
- Lagrangian-based method named Progressive Hedging (PH) proposed by (Rockafellar & Wets, 1991) which has already applied for stochastic network protection problems (Fan & Liu, 2010).

$$\max_{\delta^1(\xi_2^\omega),\delta^2(\xi_2^\omega),\upsilon_a(\xi_2^\omega)} \sum_{\omega\in\Omega} P_\omega F(\xi_2^\omega) \tag{27}$$

s.t.

$$F(\xi_2^\omega) = \sum_{k\in K} Mw_k^R R_k^D - w_k^R (R_k^{TT})^{-1} \tag{28}$$

$$\boldsymbol{\delta^1(\xi_2^\omega) - \delta^{1,z}} = 0, \forall\omega\in\Omega \tag{29}$$

$$+ \text{ other constraints}$$

- Where $P_\omega = P(\{\omega\}), \forall\omega\in\Omega$ is the probability of scenario ω taking place in the probability space
- (ξ_2^ω) is replaced with (ξ_2) in all variables to indicate that the quantities are scenario dependant.

$$\boldsymbol{\delta^1(\xi_2^\omega)} = \boldsymbol{\delta^1(\xi_2^{\omega'})}, \forall \omega \in \Omega, \forall \omega' \in \Omega, \omega' \neq \omega \tag{30}$$

Which is equivalent to the following:

$$\boldsymbol{\delta^1(\xi_2^\omega)} - \boldsymbol{\delta^{1,z}} = 0, \forall \omega \in \Omega \tag{31}$$

$\boldsymbol{\delta^{1,z}}$ is the vector of free variables.

Why needed?

To assure that the first stage decision variables are not scenario-dependant.

Admissible solution systems satisfy constraints for all scenarios!

1. **Initialisation.** Set an index k equal to 0.
2. Solve
   $\delta^1(\xi_2^\omega)^{(k)} := argmax_{\delta^1(\xi_2^\omega),\delta^2(\xi_2^\omega)} F(\xi_2^\omega) : (\delta^1(\xi_2^\omega), \delta^2(\xi_2^\omega)) \in M^\omega, \forall\omega \in \Omega.$
3. $\delta^{1,z^{(k)}} := \sum_{\omega\in\Omega} P_\omega \delta^1(\xi_2^\omega)^{(k)}$
4. $\Theta^{\omega,1^{(k)}} := r(\delta^1(\xi_2^\omega)^{(k)} - \delta^{1,z^{(k)}}), \forall\omega \in \Omega$
5. **Iteration update.** $k := k + 1$
6. **Decomposition.**
   $\delta^1(\xi_2^\omega)^{(k)} := argmax_{\delta^1(\xi_2^\omega{}_2),\delta^2(\xi_2^\omega)} F(\xi_2^\omega) - [\Theta^{\omega(k-1)}]^T.\delta^1(\xi_2^\omega) - \frac{r}{2}||\delta^1(\xi_2^\omega) - \delta^{1,z^{(k-1)}}||^2 : (\delta^1(\xi_2^\omega), \delta^2(\xi_2^\omega)) \in M^\omega, \forall\omega \in \Omega$
7. **Aggregation** $\delta^{1,z^{(k)}} := \sum_{\omega\in\Omega} P_\omega \delta^1(\xi_2^\omega)^{(k)}$
8. **Weight update.**$\Theta^{\omega,1^{(k)}} := \Theta^{\omega,1^{(k-1)}} + r(\delta^1(\xi_2^\omega)^{(k)} - \delta^{1,z^{(k)}}), \forall\omega \in \Omega$
9. **Termination criterion.** If $\sum_{\omega\in\Omega} P_\omega||\delta^1(\xi_2^\omega)^{(k)} - \delta^{1,z^{(k)}}|| \leq \epsilon$, then go to 5, else, stop.

- Where Θ is the vector of dual variables for the constraints in 31 (Weights) and r is the penalty value.
- $M^\omega, \forall\omega \in \Omega$ is the set of all the feasible solutions for scenario ω that meets the constraints of the problem

Outline

Numerical experiment
Network

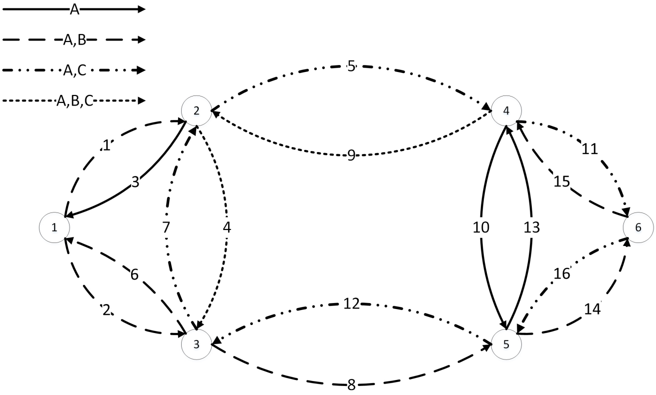


Figure: Road network for assessment.

Numerical experiment
Unit costs of actions

Link	Actions				
	Pre-disaster actions			Post-response	
	Expansion - per unit of expansion	Fortification - in percentage		response - per unit of response	
	b^e_a	$b^{ret}_{a,1}$ (per 1%)	$b^{ret}_{a,2}$,(per 1%)	$b^{rec}_{a,1}$ $(t^{rec}_{a,z},p^{rec}_{a,z})$	$b^{rec}_{a,2}$ $(t^{rec}_{a,z},p^{rec}_{a,z})$
1	0	0.4	0.5	6 (3,1)	0
2	0	0.6	0	7(4,2)	0
3	2	0.5	0	0	6(3,1)
4	0	0.1	0.5	3(2,1)	2(1,1)
5	0	0.8	0	0	0
6	0	0.6	0	7.5(4,2)	0
7	0	0	0	4.5(2,1)	0
8	3	0.5	0	0	4.6(3,1)
9	0	0.8	0	5(6,2)	6(5,2)
10	0	0	0.5	0	0
11	0	0	0.5	6(3,1)	10(3,2)
12	0	0	0.5	12(4,3)	0

Numerical experiment

Forthcoming natural disasters and their liklihood

Scenario ID	Hazard type	Links affected (capacity loss %)	Liklihood
A	1000-year Flood	1(100%),4(100%),10(100%),13(100%),11(100%),8(100%),9(100%),14(100%)	0.01
B	500-year Flood	1(100%),4(100%),10(100%),11(100%),8(100%),14(100%),15(100%)	0.02
C	100-year Flood	4(100%),10(100%),11(100%),8(100%),14(100%),16(100%)	0.1
D	50-year Flood	1(100%),11(100%),14(100%)	0.2
E	10-year Flood	1(100%),4(100%)	0.2
F	Landslide	4(80%),11(90%),8(70%)	0.25
G	Landslide	4(50%),13(80%),8(50%),14(60%),7(50%)	0.22

Numerical experiment

Results for demand satisfaction resilience

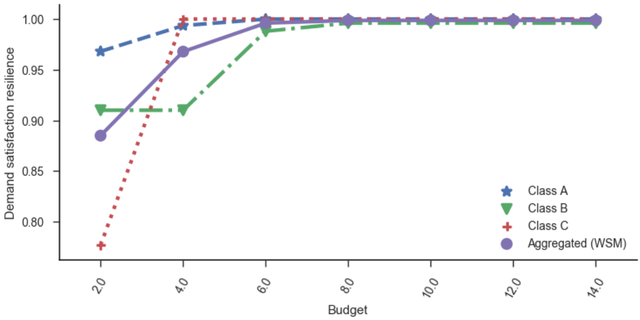


Figure: DS resilience for different vehicle classes under various budgets for the numerical experiment.

Numerical experiment

Results for travel time resilience

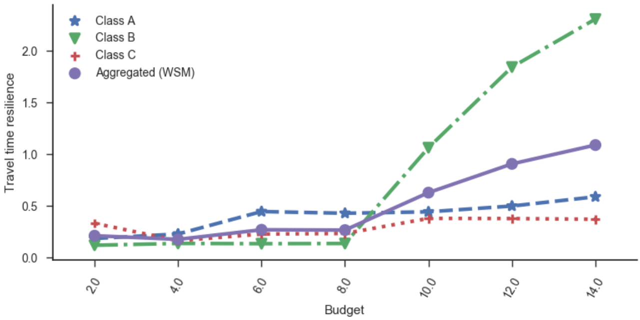


Figure: TT resilience for different vehicle classes under various budgets for the numerical experiment.

Summary

- A bi-level NDP used to optimise resilience.
- Problem converted to SMPEC
- Schur's decomposition and SOS type two variables used for linearisation and linear reformulation.
- Progressive hedging applied for solving the reformulated stochastic MILP.

For Further Reading I

Asadabadi, A., & Miller-Hooks, E. (2017). Optimal transportation and shoreline infrastructure investment planning under a stochastic climate future. *Transportation Research Part B: Methodological*, *100*, 156–174.

Chen, A., & Xu, X. (2012). Goal programming approach to solving network design problem with multiple objectives and demand uncertainty. *Expert Systems with applications*, *39*(4), 4160–4170.

Fan, Y., & Liu, C. (2010). Solving stochastic transportation network protection problems using the progressive hedging-based method. *Networks and Spatial Economics*, *10*(2), 193–208.

For Further Reading II

Faturechi, R., & Miller-Hooks, E. (2014a). Measuring the performance of transportation infrastructure systems in disasters: A comprehensive review. *Journal of infrastructure systems*, *21*(1), 04014025.

Faturechi, R., & Miller-Hooks, E. (2014b). Travel time resilience of roadway networks under disaster. *Transportation research part B: methodological*, *70*, 47–64.

Hasan, M., & Karimi, I. (2010). Piecewise linear relaxation of bilinear programs using bivariate partitioning. *AIChE journal*, *56*(7), 1880–1893.

Kaviani, A., Thompson, R. G., Rajabifard, A., & Sarvi, M. (2018). A model for multi-class road network recovery scheduling of regional road networks. *Transportation*, 1–35.

For Further Reading III

Liu, H., & Wang, D. Z. (2015). Global optimization method for network design problem with stochastic user equilibrium. *Transportation Research Part B: Methodological*, *72*, 20–39.

Rockafellar, R. T., & Wets, R. J.-B. (1991). Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, *16*(1), 119–147.

Siddiqui, S., & Gabriel, S. A. (2013). An sos1-based approach for solving mpecs with a natural gas market application. *Networks and Spatial Economics*, *13*(2), 205–227.

Silva, T. L., & Camponogara, E. (2014). A computational analysis of multidimensional piecewise-linear models with applications to oil production optimization. *European Journal of Operational Research*, *232*(3), 630–642.

For Further Reading IV

Unnikrishnan, A., & Lin, D.-Y. (2012). User equilibrium with recourse: continuous network design problem. *Computer-Aided Civil and Infrastructure Engineering*, *27*(7), 512–524.

Van Slyke, R. M., & Wets, R. (1969). L-shaped linear programs with applications to optimal control and stochastic programming. *SIAM Journal on Applied Mathematics*, *17*(4), 638–663.

Wang, D. Z., & Lo, H. K. (2010). Global optimum of the linearized network design problem with equilibrium flows. *Transportation Research Part B: Methodological*, *44*(4), 482–492.

Wang, D. Z., & Xie, D.-F. (2016). Reliable transportation network design considering uncertain demand variability. *International Journal of Sustainable Transportation*, *10*(8), 752–763.

For Further Reading V

Wu, J. H., Florian, M., & He, S. (2006). An algorithm for multi-class network equilibrium problem in pce of trucks: application to the scag travel demand model. *Transportmetrica*, *2*(1), 1–9.

Yang, H., & H. Bell, M. G. (1998). Models and algorithms for road network design: a review and some new developments. *Transport Reviews*, *18*(3), 257–278.

Yin, Y., & Ieda, H. (2002). Optimal improvement scheme for network reliability. *Transportation Research Record: Journal of the Transportation Research Board*(1783), 1–6.

Yin, Y., Madanat, S. M., & Lu, X.-Y. (2009). Robust improvement schemes for road networks under demand uncertainty. *European Journal of Operational Research*, *198*(2), 470–479.