Optimal Control of a UAV in Search & Rescue Operations

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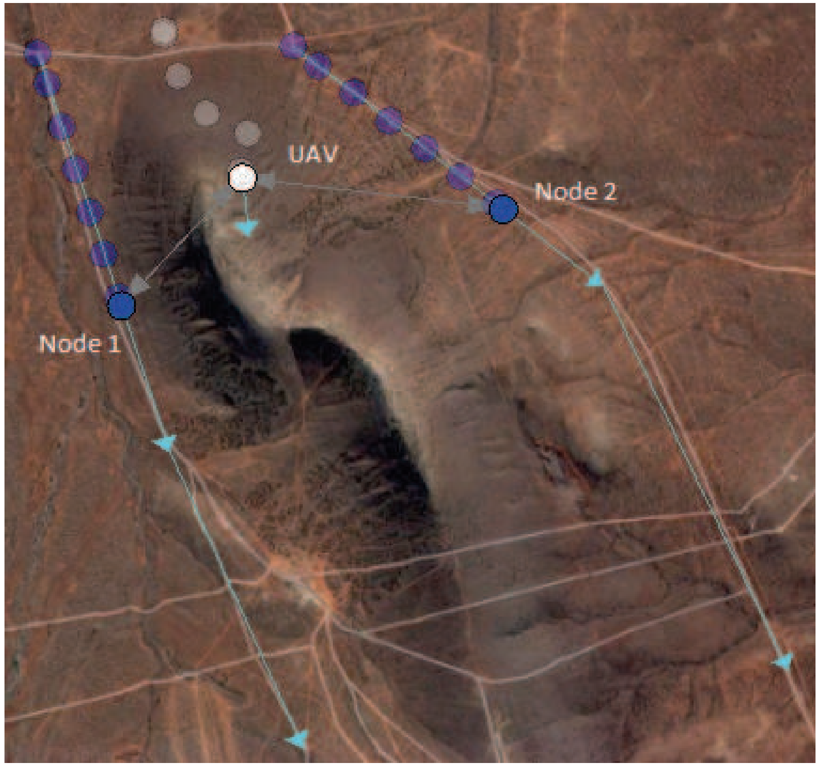
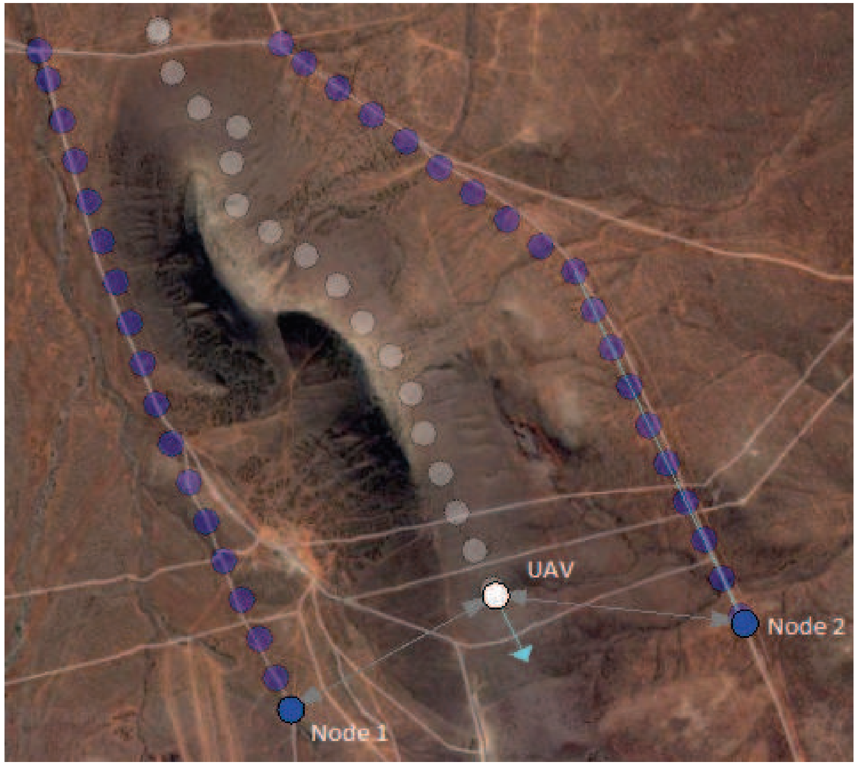
Outline

A Motivation



(a) The autonomous UAV trying to maximize the SNR of the weakest link.

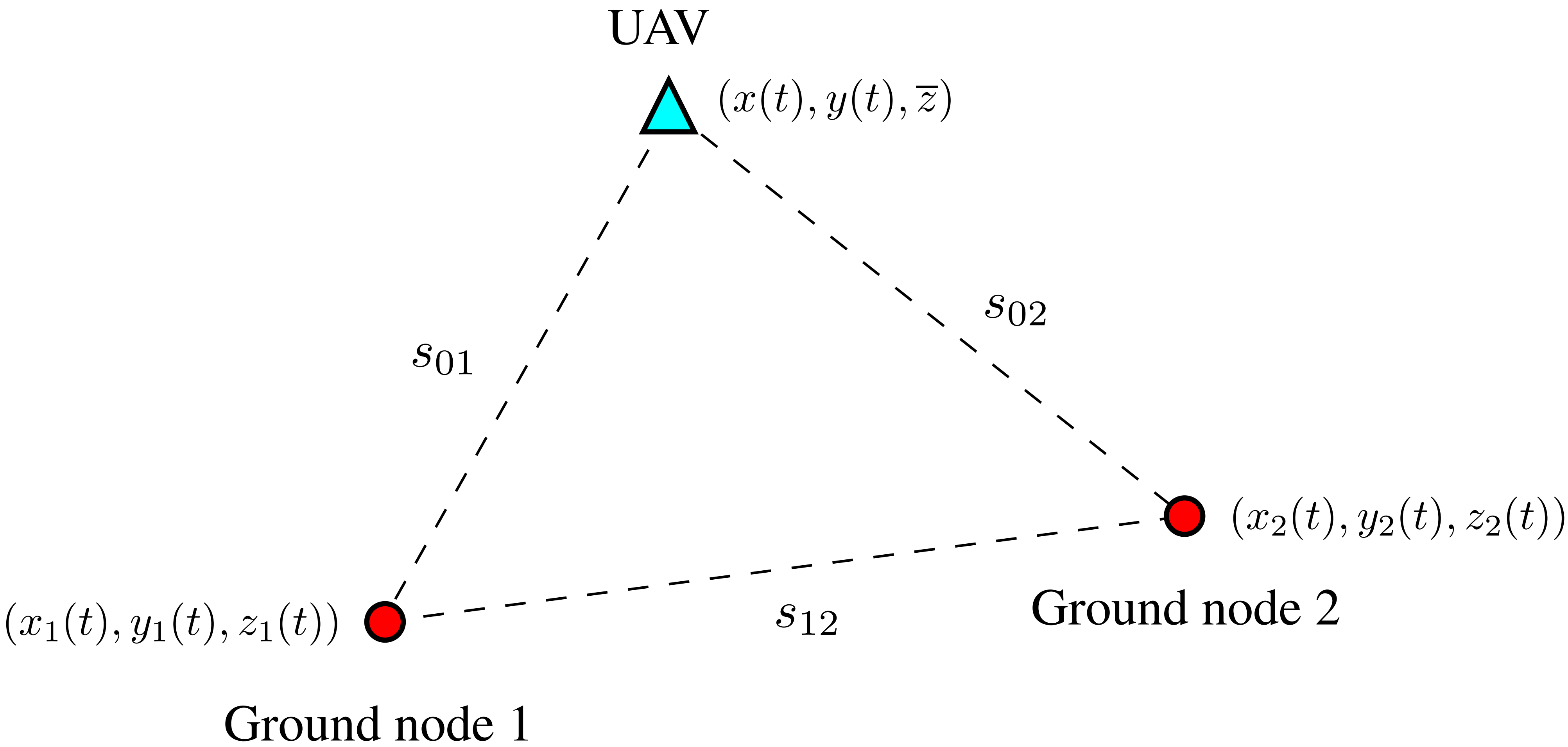


(b) The autonomous UAV maintains connectivity throughout the mission.

Fig. 3: Emulation of Tactical Scenario 1.

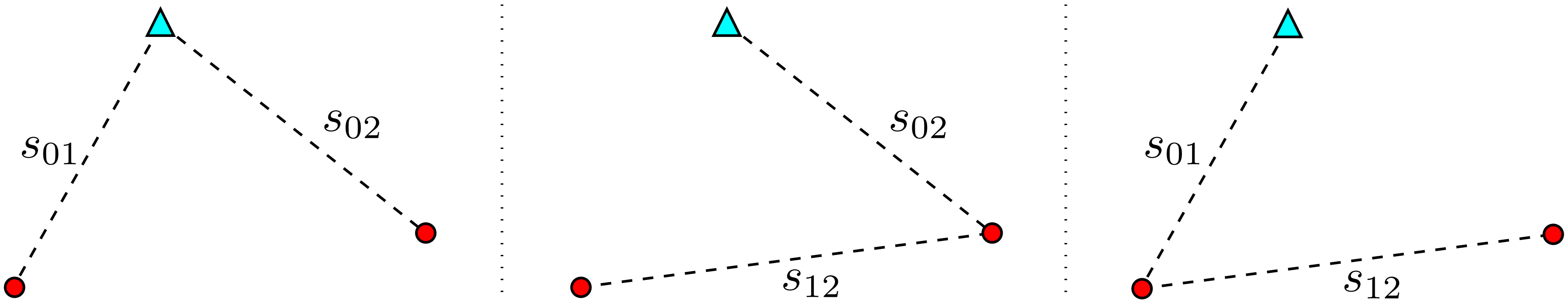
Ref.: K.-P. Hui, D. Phillips and A. Kekirigoda, *Beyond line-of-sight range extension with OPAL using autonomous unmanned aerial vehicles.* MILCOM 2017.

A 3-Node Network



Network Connection Level

In a connected network of three nodes, if the “weakest link,” i.e., the longest edge, is removed, then the network is minimally connected - if you delete another edge, the network is not connected anymore.



$$\text{NCL} \sim \frac{1}{\max\{s_{01}, s_{02}\}^p} \qquad \text{NCL} \sim \frac{1}{\max\{s_{02}, s_{12}\}^p} \qquad \text{NCL} \sim \frac{1}{\max\{s_{01}, s_{12}\}^p}$$

$\sim$: “A measure of”. Typically, $p \geq 2$

$$1/\text{NCL} \sim \max\Big\{\min\{s_{01}, s_{02}\}, \min\{s_{01}, s_{12}\}, \min\{s_{02}, s_{12}\}\Big\}^p$$

Network Connection Level

$$
\begin{aligned}
s_{01}^2(x(t), y(t), t) &= (x(t) - x_1(t))^2 + (y(t) - y_1(t))^2 + (\overline{z} - z_1(t))^2 \\
s_{02}^2(x(t), y(t), t) &= (x(t) - x_2(t))^2 + (y(t) - y_2(t))^2 + (\overline{z} - z_2(t))^2 \\
s_{12}^2(t) &= (x_1(t) - x_2(t))^2 + (y_1(t) - y_2(t))^2
\end{aligned}
$$

$$
\begin{aligned}
\varphi(x, y, t) := \max \Big\{ & \min\{s_{01}(x, y, t), s_{02}(x, y, t)\}, \\
& \min\{s_{01}(x, y, t), s_{12}(t)\}, \\
& \min\{s_{02}(x, y, t), s_{12}(t)\} \Big\}
\end{aligned}
$$

maximize a meas. of NCL $\equiv$ minimize a meas. of $\varphi(x, y, t)$

Optimization of Network Connection Level

$$
\text{(NCL)} \left\{ \begin{array}{lll}
\displaystyle\min_{x,y,\theta,u,v} & \displaystyle\int_0^{t_f} \varphi(x(t),y(t),t)\,dt & \left(\text{or } \displaystyle\max_{0\le t\le t_f} \varphi(x(t),y(t),t) \right) \\
\text{s.t.} & \dot{x}(t) = v(t)\,\cos\theta(t)\,, & x(0) = x_0\,, \\
 & \dot{y}(t) = v(t)\,\sin\theta(t)\,, & y(0) = y_0\,, \\
 & \dot{\theta}(t) = u(t)\,, & \theta(0) = \theta_0\,, \\
 & & v_{\min} \le v(t) \le v_{\max}\,,
\end{array} \right.
$$

where $\dot{x} = dx/dt$, etc. Problem (NCL) is an *optimal control problem*. Here, $x(t)$, $y(t)$ and $\theta(t)$ are the state variables and $u(t)$ and $v(t)$ the control variables.

$(x(t),y(t))$: the coordinates, and $\theta(t)$: the orientation, of the UAV.

Since φ is non-smooth and non-convex, Problem (NCL) is non-smooth and non-convex. This makes the problem *non-standard*. **Difficult to solve!**

Num. Methods for Optimal Control Problems

Differentiable problems

- Indirect methods
  - Optimize-then-discretize: TPBVP arising from optimality conditions is solved by
    (i) Discretization + large-scale optimization (e.g., using AMPL+Ipopt)
    or
    (ii) Shooting methods

- Direct methods
  - Discretize-then-optimize: Discretization + large-scale optimization (e.g., using AMPL+Ipopt)

Non-differentiable (or nonsmooth) problems

Not much is available in the literature.

Solution of Problem (NCL)

(1) We discretized Problem (NCL), using Euler discretization, and then employed the discretize-then-optimize approach.

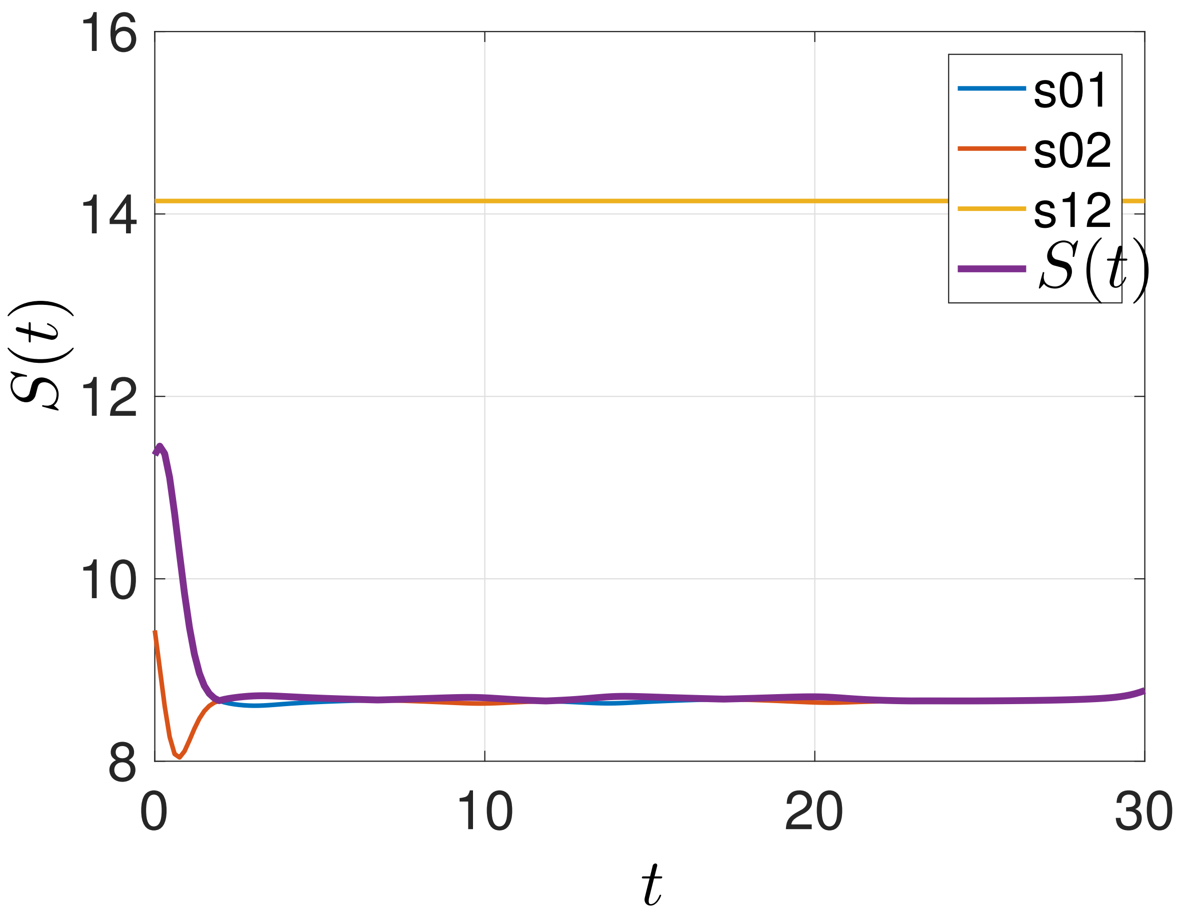
(2) Although Problem (NCL) is non-smooth, we utilized a differentiable solver, which produced at least some approximate solutions – to our pleasant surprise!

(3) These solutions should be met with caution. The ideal would be to employ a nonsmooth technique, such as the deflected subgradient (DSG) method (Burachik & Kaya, 2006–present), employing a nonsmooth software, such as SolvOpt, for the subproblems of the DSG method.

Experiments with $N = 200$ subdivisions. $v_{\min} = 0$ or 0.5; $v_{\max} = 10$.

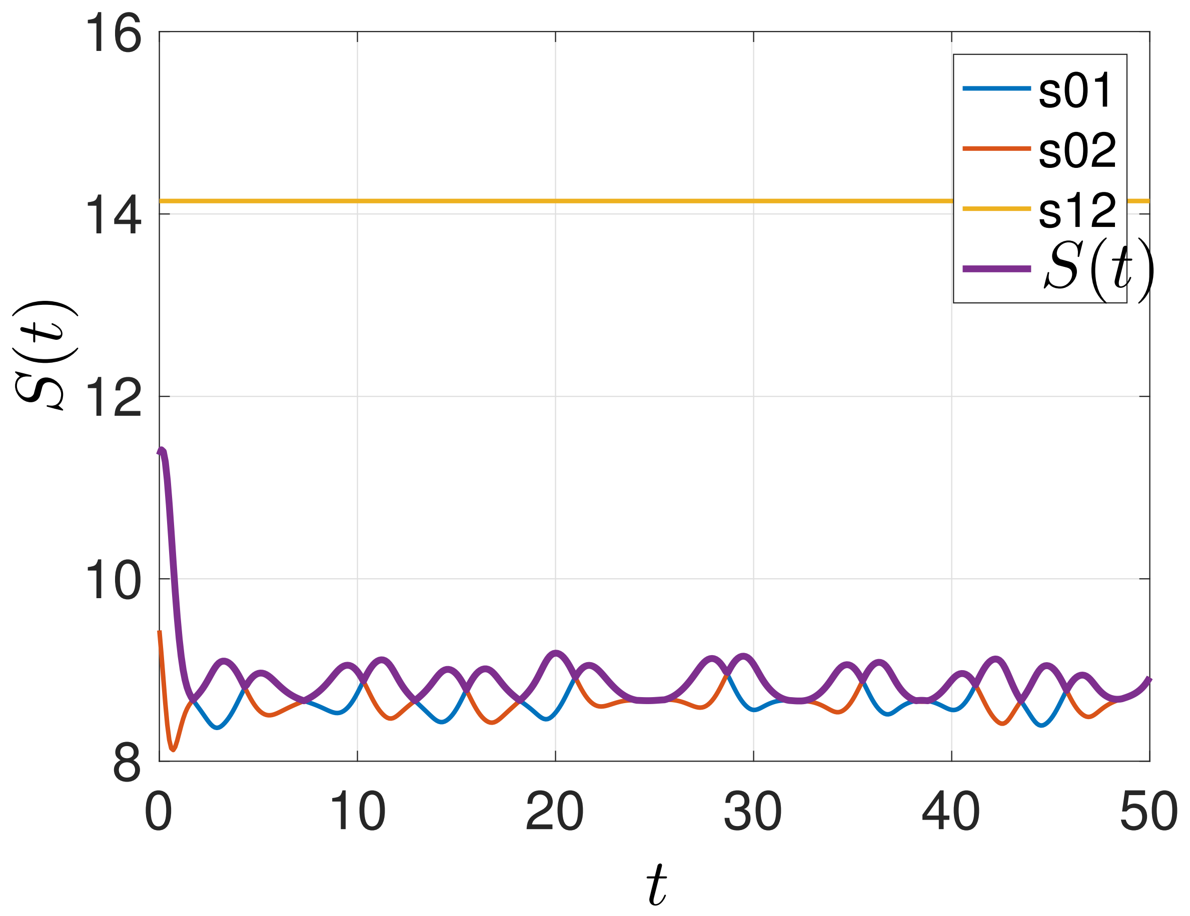
Solution of Problem (NCL)

Two fixed ground nodes with $v_{\min} = 0$ (View the movie!)



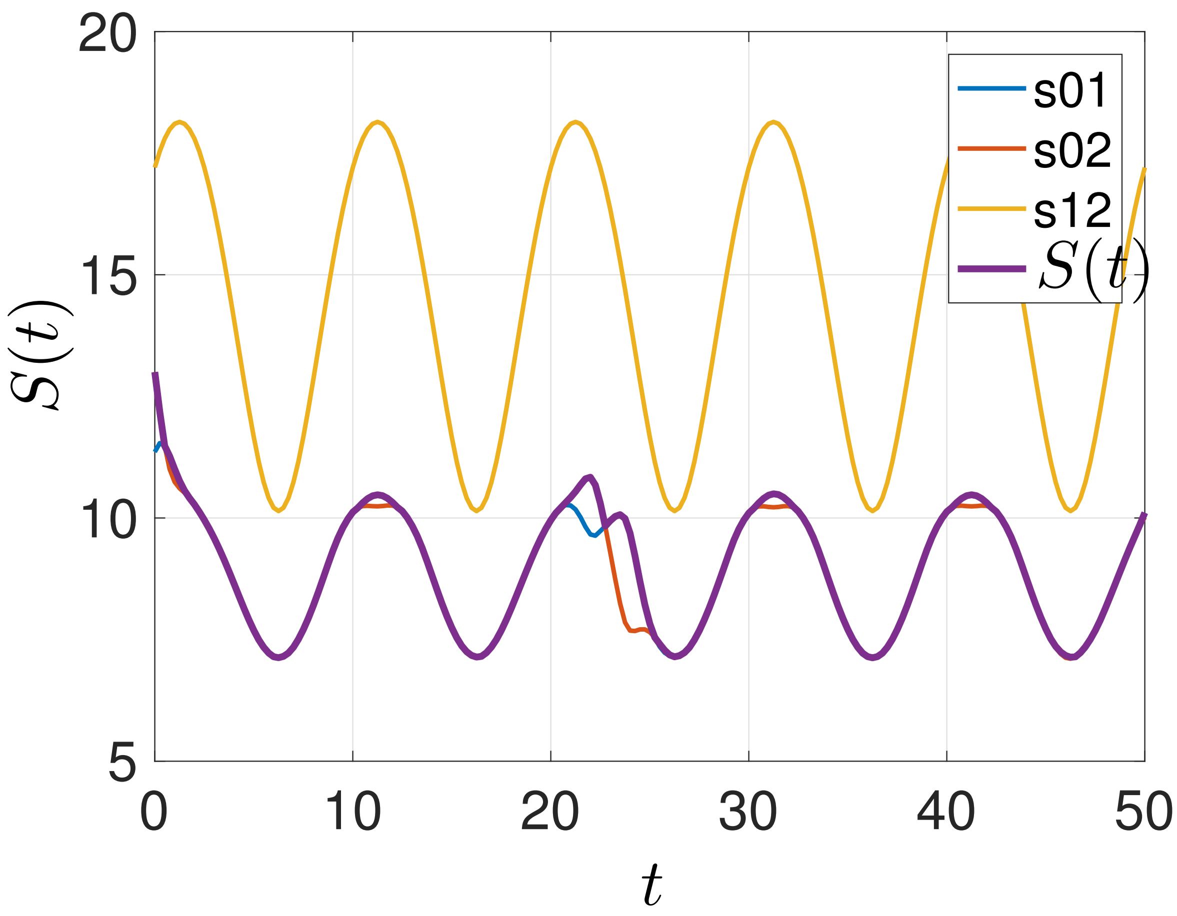
Solution of Problem (NCL)

Two fixed ground nodes with $v_{\min} = 0.5$ (View the movie!)



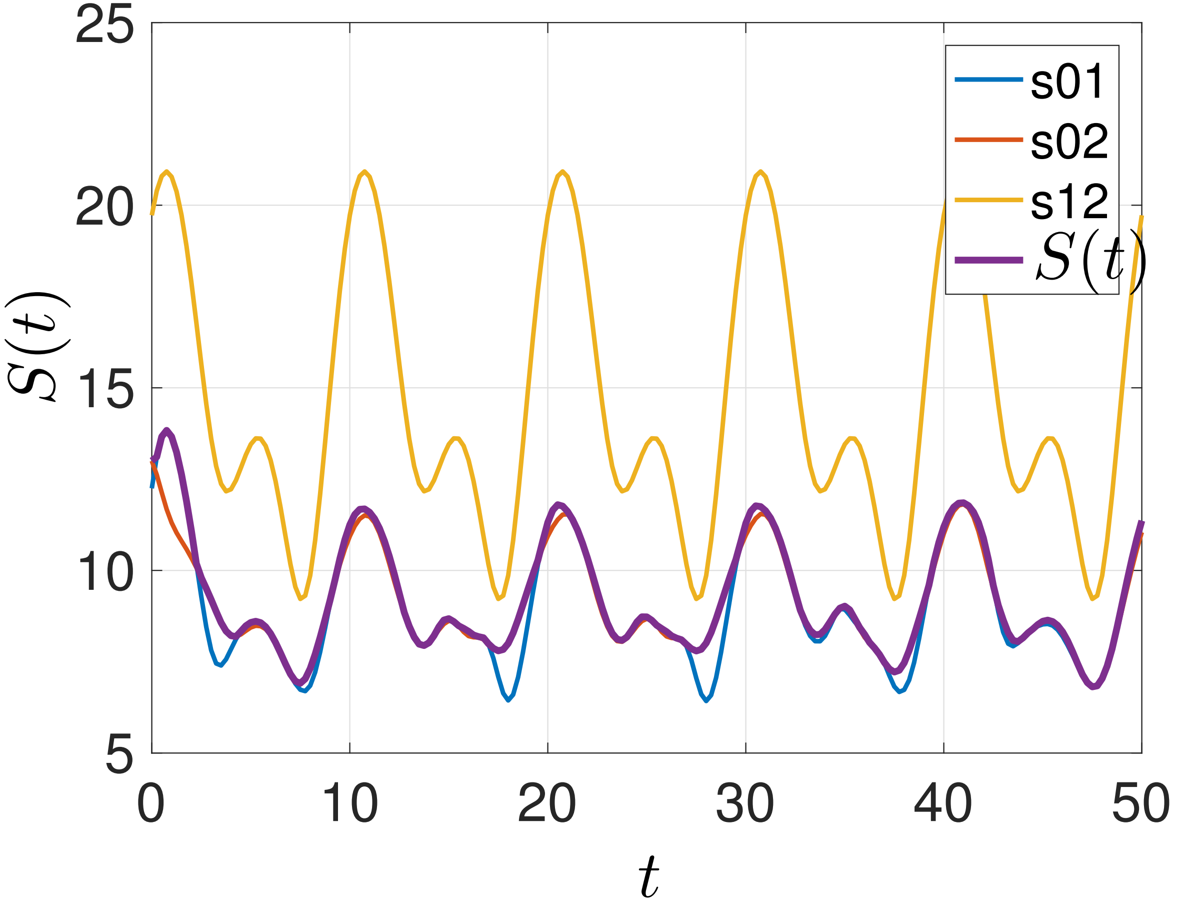
Solution of Problem (NCL)

1 circular-moving 1 fixed node with $v_{\min} = 0.5$ (View the movie!)



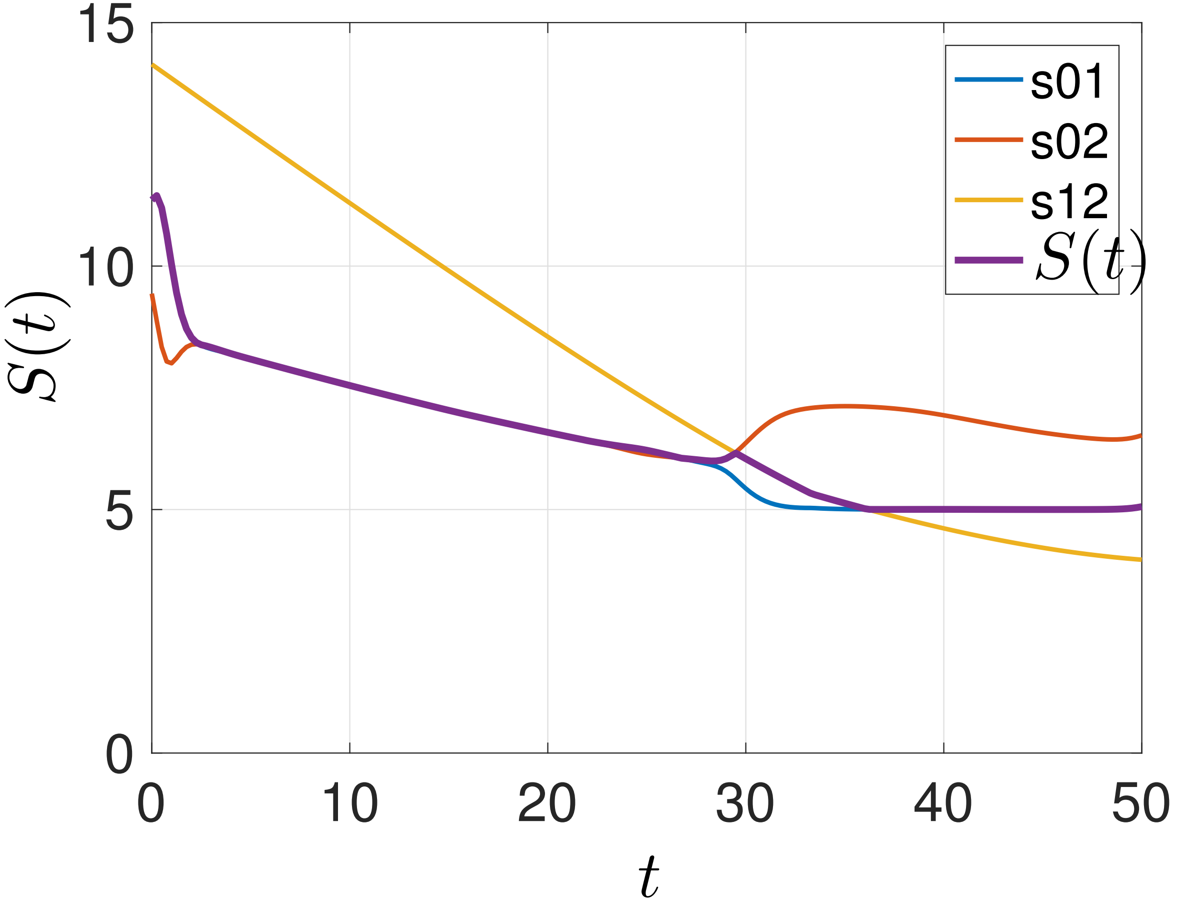
Solution of Problem (NCL)

Two circular-moving nodes with $v_{\min} = 0.5$ (View the movie!)



Solution of Problem (NCL)

Two linear-moving nodes with $v_{\min} = 0$ (View the movie!)



Solution of Problem (NCL) Under Uncertainty

(1) Consider the case when $(x(t), y(t))$ is only known with uncertainty, say $x(t)$ and $y(t)$ come from a normal distribution.

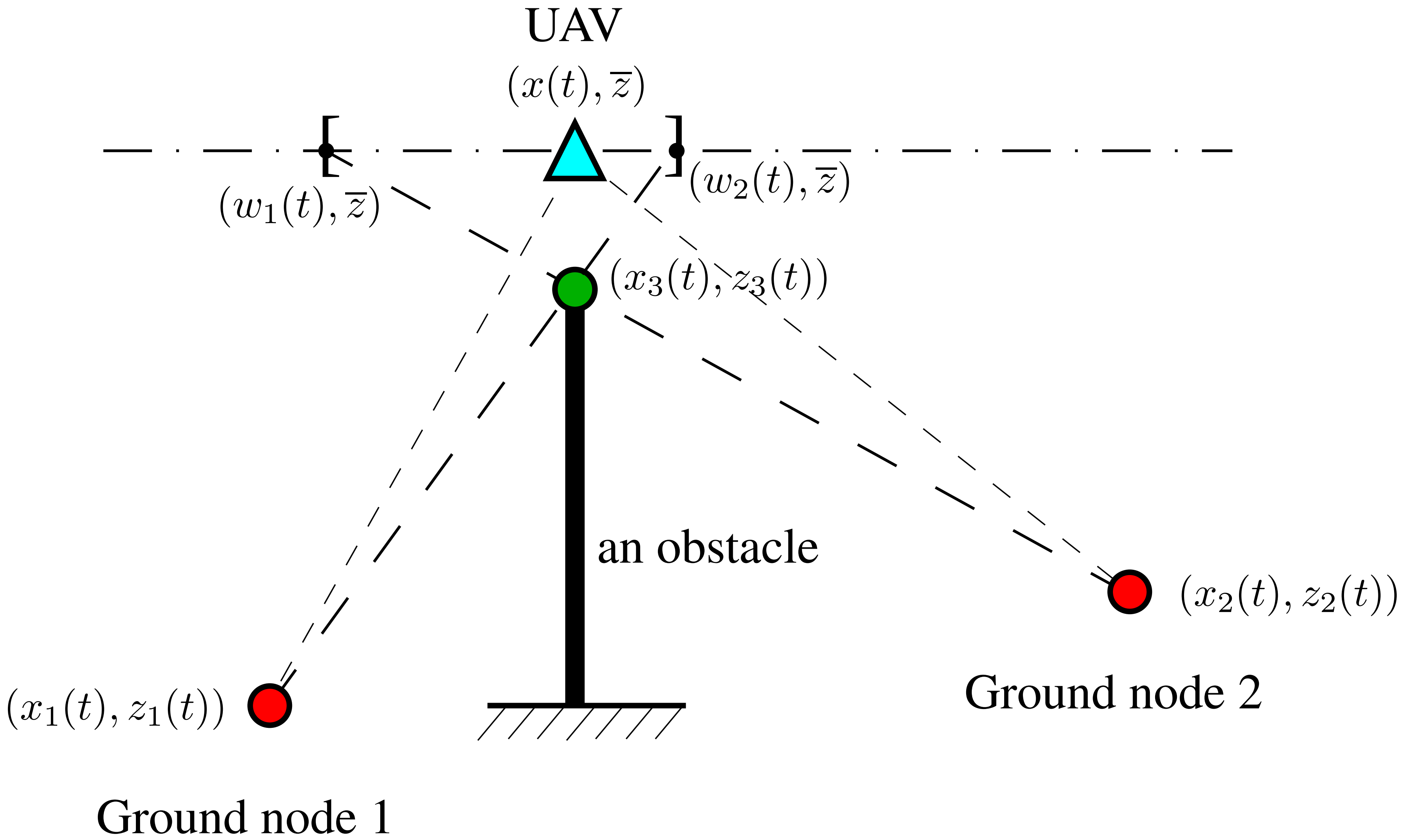
(2) We implemented these uncertainties in the model and computed the optimal trajectories under these uncertainties.

(3) The computed trajectories look robust, in the sense that they don't differ in quality from those in absence of uncertainty.

(View more movies!)

Confidence in Connectivity

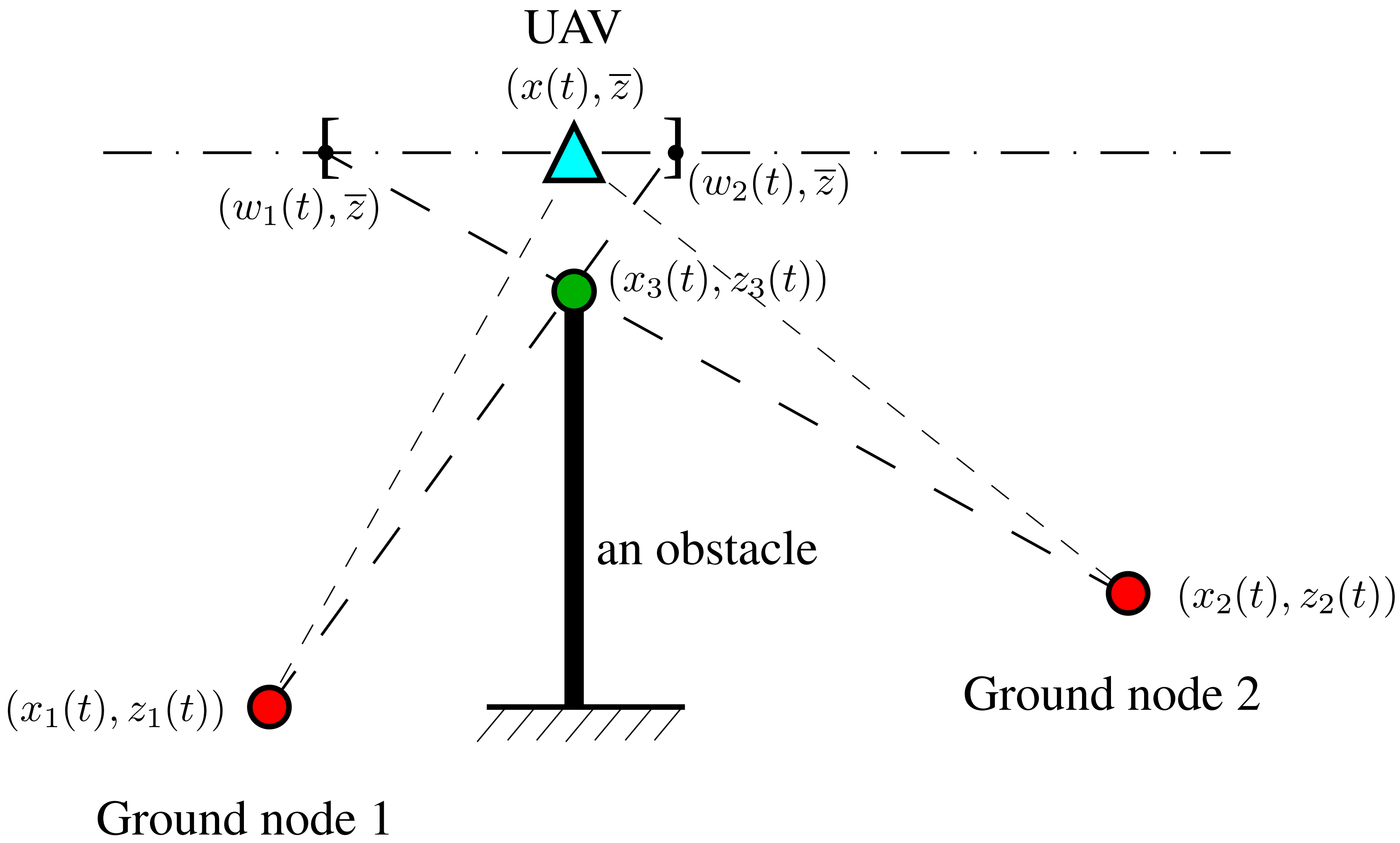
A (further) simplified planar model, but with a constraint:



Ground nodes 1 and 2 are no longer directly connected!

Confidence in Connectivity

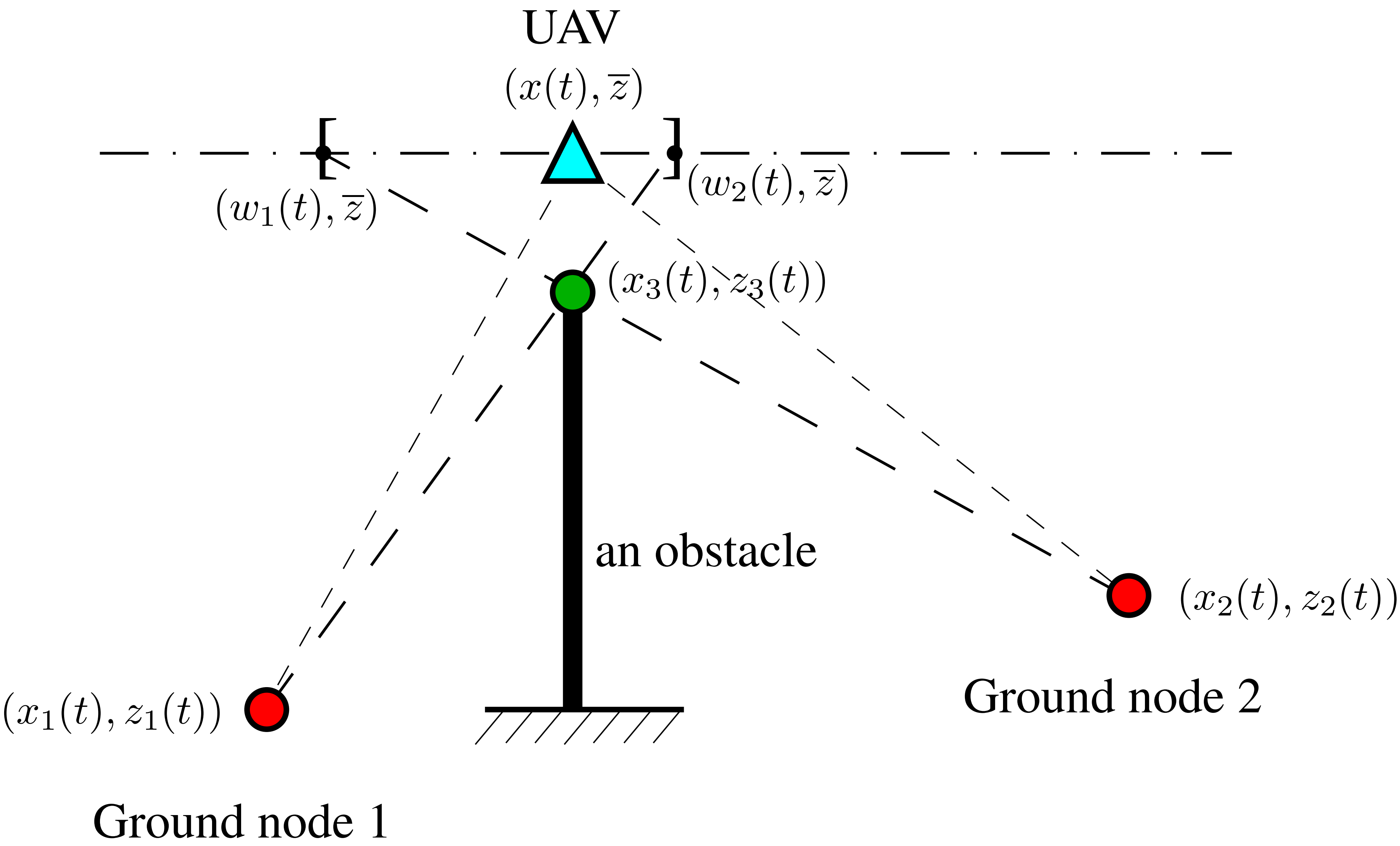
A (further) simplified planar model, but with a constraint:



In maximization of NCL, $x(t)$ is constrained to be in $[w_1(t), w_2(t)]$.

Confidence in Connectivity

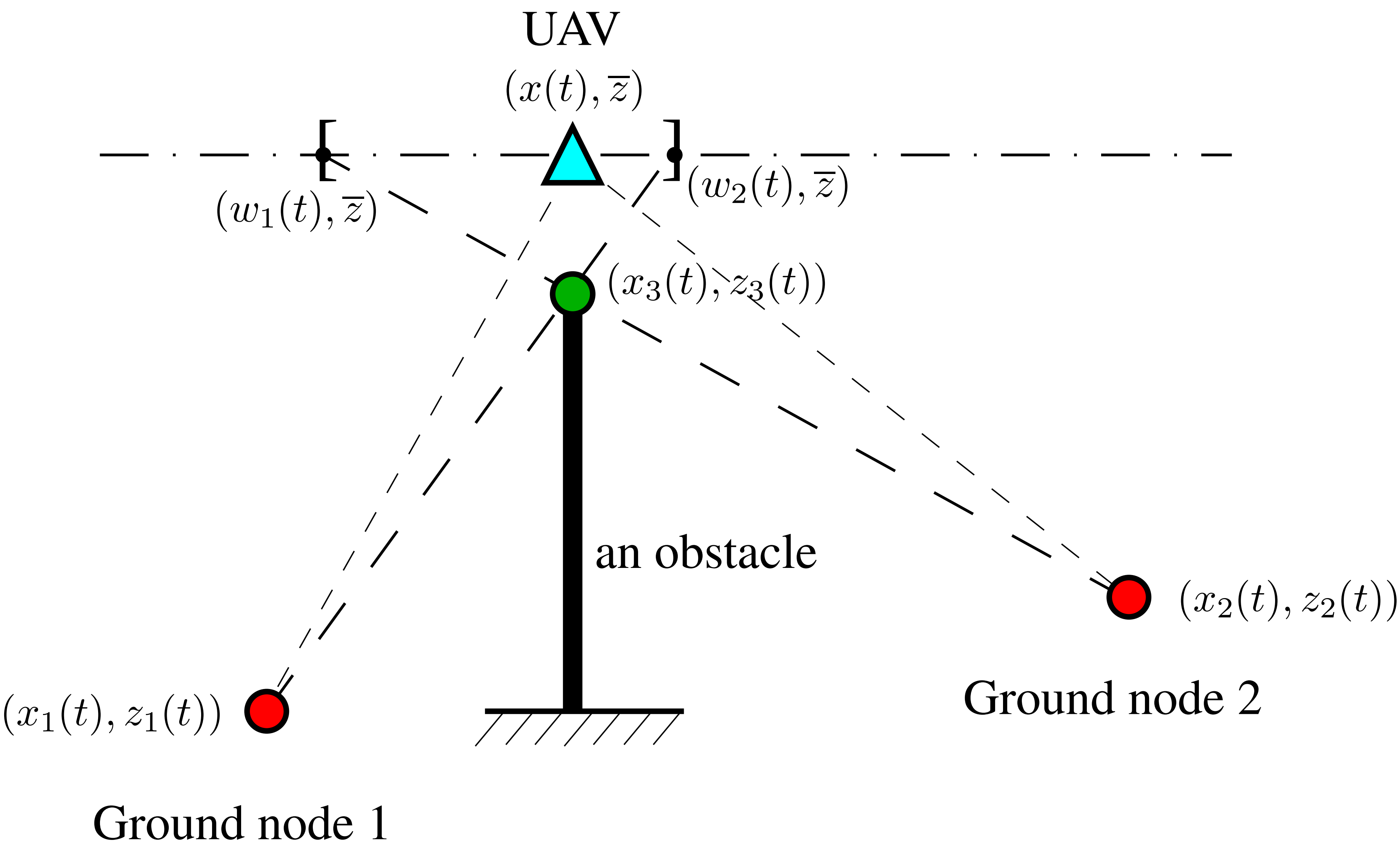
A (further) simplified planar model, but with a constraint:



Suppose that the knowledge of $x_1(t)$ and $x_2(t)$ is uncertain.

Confidence in Connectivity

A (further) simplified planar model, but with a constraint:



Define the *level of confidence* that UAV can sight both nodes 1 & 2.

Confidence in Connectivity

- Let $X_1(t)$ and $X_2(t)$ be random variables at time t. Then $W_1(t)$ and $W_2(t)$ will be associated random variables at time t by the given geometry.

- $Pr(W_1(t) \leq x(t))$: The probability that the UAV position $x(t)$ is to the right of $w_1(t)$, i.e. the UAV can sight node 2.

- $Pr(W_2(t) \geq x(t))$: The probability that the UAV position $x(t)$ is to the left of $w_2(t)$, i.e. the UAV can sight node 1.

- Define the confidence level function γ as

$$\gamma(x(t), t) = \min\{Pr(W_1(t) \leq x(t)), Pr(W_2(t) \geq x(t))\}.$$

Optimization of Confidence Level

The problem of maximizing the probability that UAV is in the interval $[w_1(t), w_2(t)]$, i.e., the probability that UAV sights both nodes 1 & 2, as

$$
(\mathrm{CL}) \begin{cases} \displaystyle\max_{x,v,u} & \displaystyle\int_0^{t_f} \gamma(x(t),t)\,dt \qquad \left(\text{or } \max_{0\le t\le t_f} \gamma(x(t),t)\right) \\ \text{s.t.} & \dot{x}(t) = v(t)\,, \qquad (x(0) = x_0)\,, \\ & \dot{v}(t) = u(t) \qquad (v(0) = v_0)\,, \\ & \qquad v_{\min} \le v(t) \le v_{\max}\,. \end{cases}
$$

Problem (CL) is another optimal control problem, with $x(t)$ and $v(t)$ as the state variables and $u(t)$ the control variable.

Since γ is non-smooth and non-convex, Problem (CL) is non-smooth and non-convex. This is also **difficult to solve** just like Problem (NCL)!

Solution of Problem (CL)

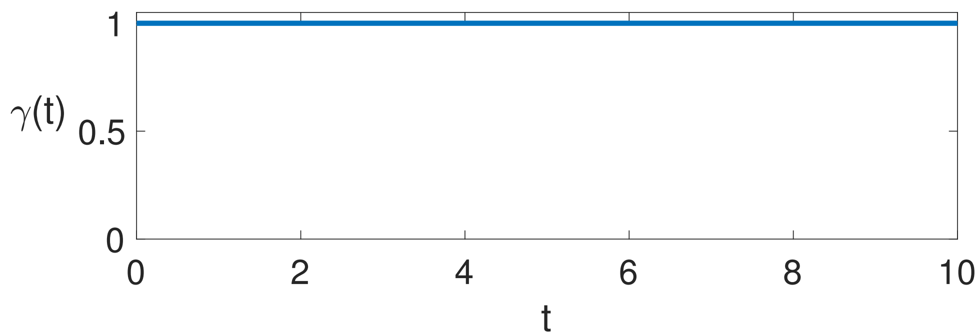
- As in the case of Problem (NCL), we Euler-discretized Problem (CL) and then used the discretize-then-optimize approach; utilized a differentiable solver to get some reasonably approximate solutions!

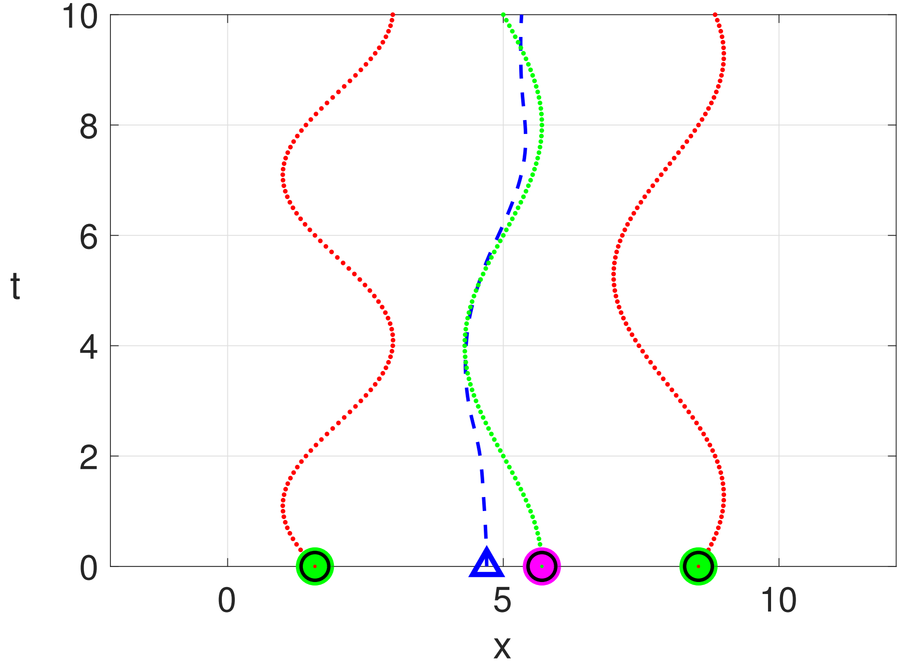
- We carried out experiments with $N = 100$ subdivisions, for comfortable viewing of the movies.

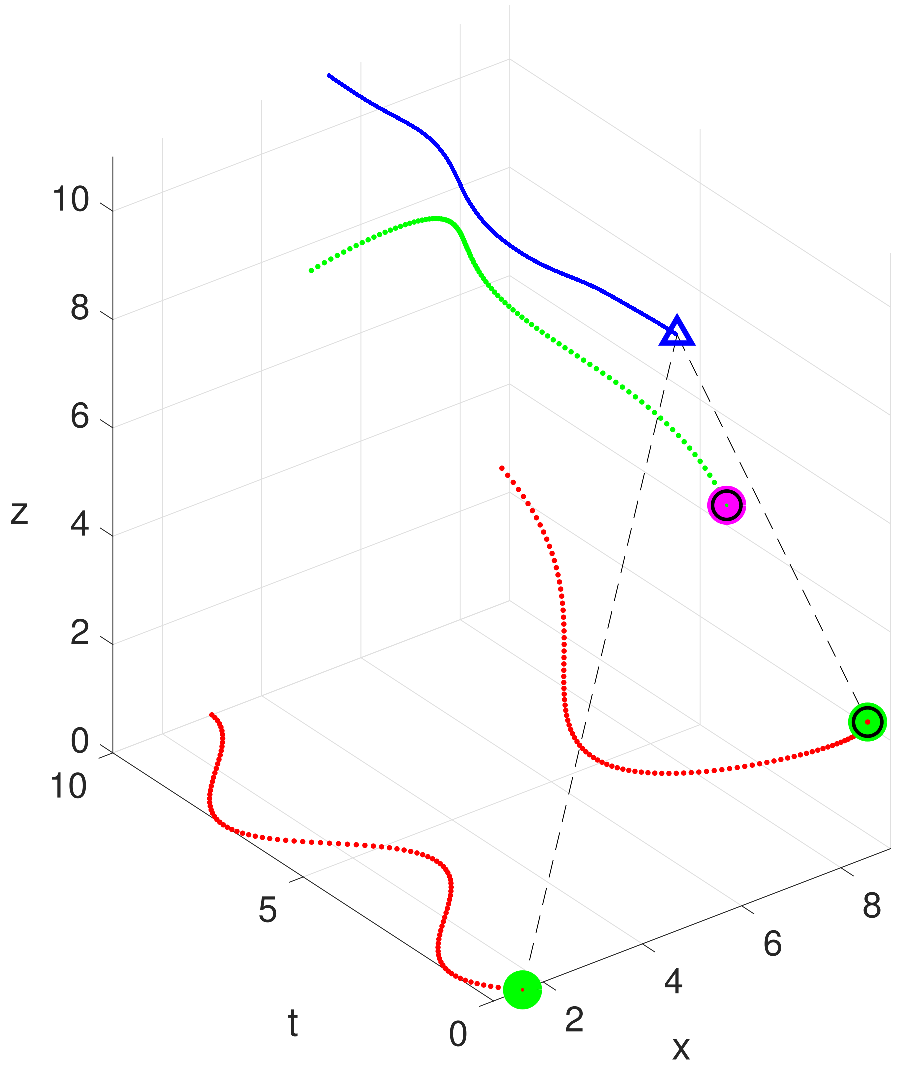
- We assumed that $X_1(t)$ and $X_2(t)$ come from the uniform distributions, $\mathcal{U}(\mu_1(t)-1, \mu_1(t)+1)$ and $\mathcal{U}(\mu_2(t)-1, \mu_2(t)+1)$, respectively, and $\mu_i(t)$ are the means at time t, given as some combination of sinusoids and parabolas, for the experimentation sake.

Solution of Problem (CL)

Full confidence (View the movie!)

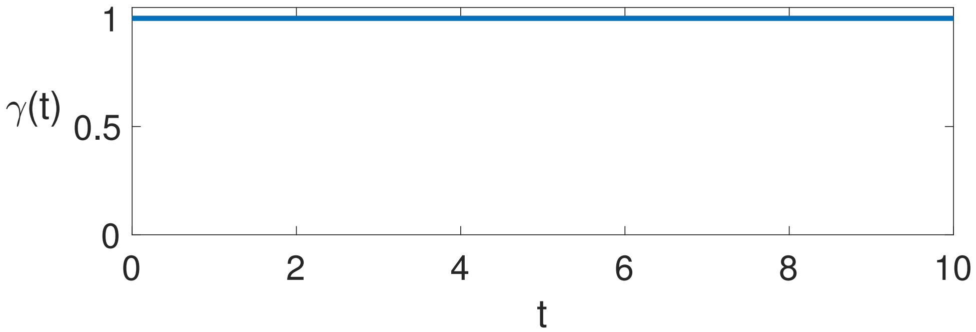


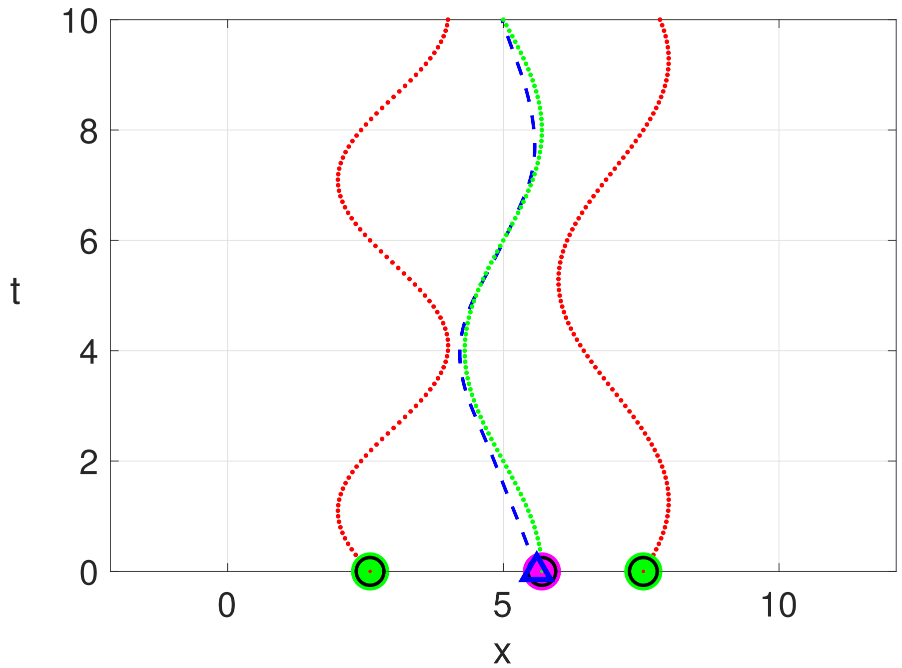


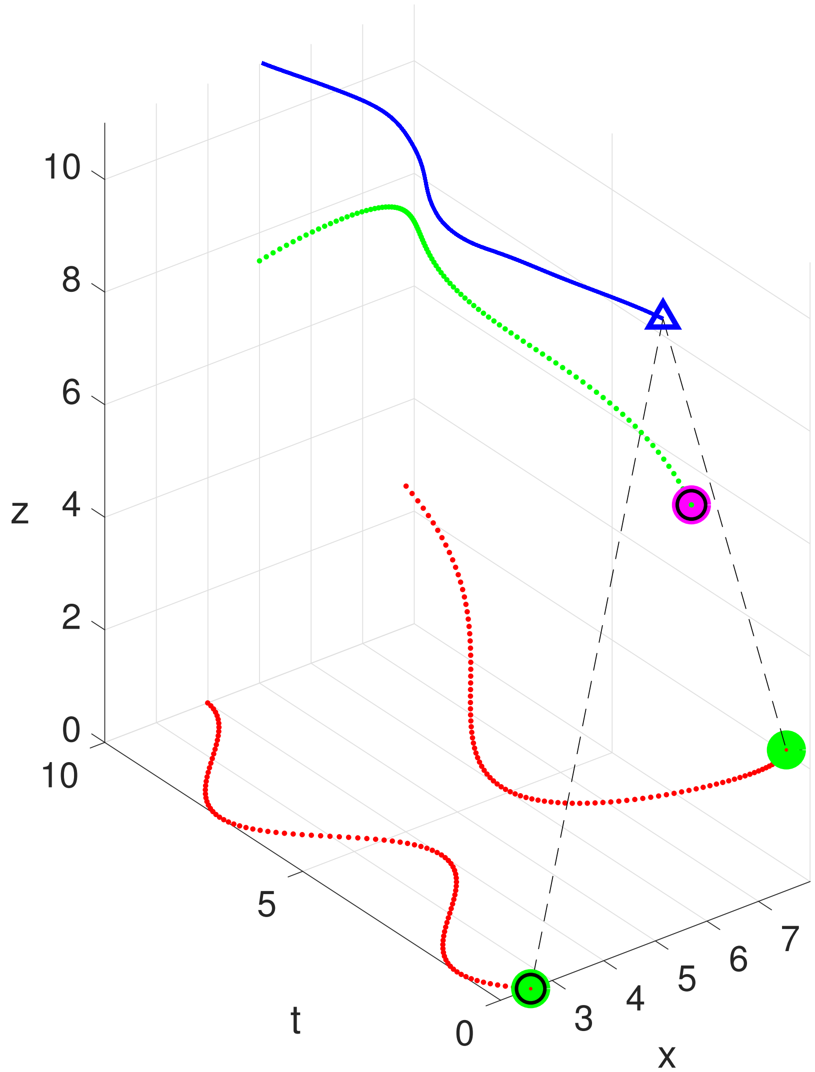


Solution of Problem (CL)

Full confidence - but more difficult (View the movie!)

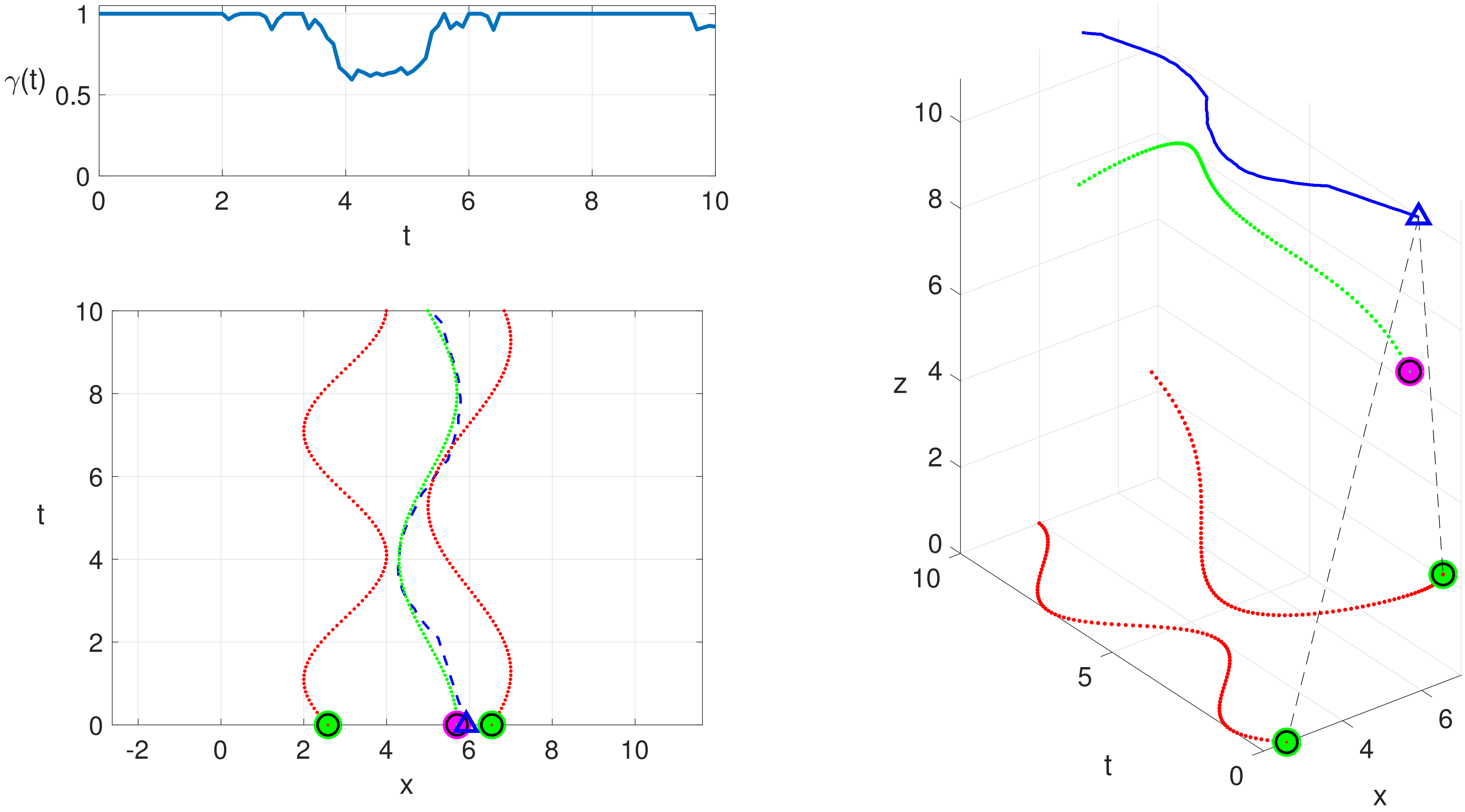






Solution of Problem (CL)

Partial confidence (View the movie!)



Simultaneous Optimization of NCL and CL

We now pose the problem of multi-objective optimal control of NCL and CL as

$$
\text{(MOP)} \left\{
\begin{array}{lll}
\displaystyle \min_{x,v,u} & \multicolumn{2}{l}{\displaystyle \left[-\int_0^{t_f} \gamma(x(t),t)\,dt,\ \int_0^{t_f} \tilde{\varphi}(x(t),t)\,dt \right]} \\
\text{s.t.} & \dot{x}(t) = v(t)\,, & (x(0) = x_0)\,, \\
 & \dot{v}(t) = u(t)\,, & (v(0) = v_0)\,, \\
 & & v_{\min} \le v(t) \le v_{\max}\,.
\end{array}
\right.
$$

The function $\tilde{\varphi}(x(t),t)$ associated with NCL, similarly to $\varphi(x(t),t)$, is defined for this simpler case as

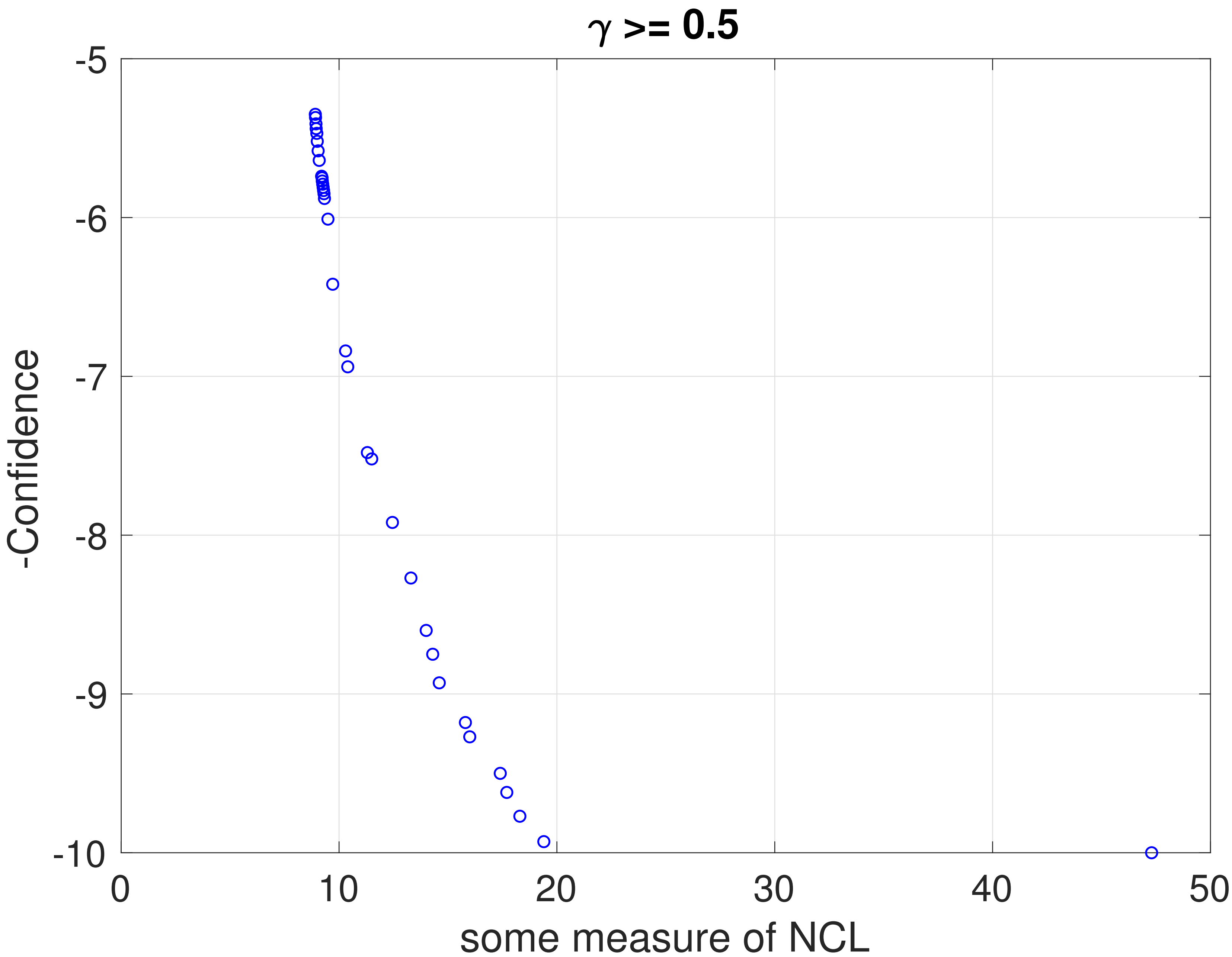
$$
\tilde{\varphi}(x(t),t) := \left[(x(t)-x_1(t))^2 + (\overline{z}-z_1(t))^2\right]^2 - \left[(x(t)-x_2(t))^2 + (\overline{z}-z_2(t))^2\right]^2
$$

Scalarization for Multi-objective Optimization

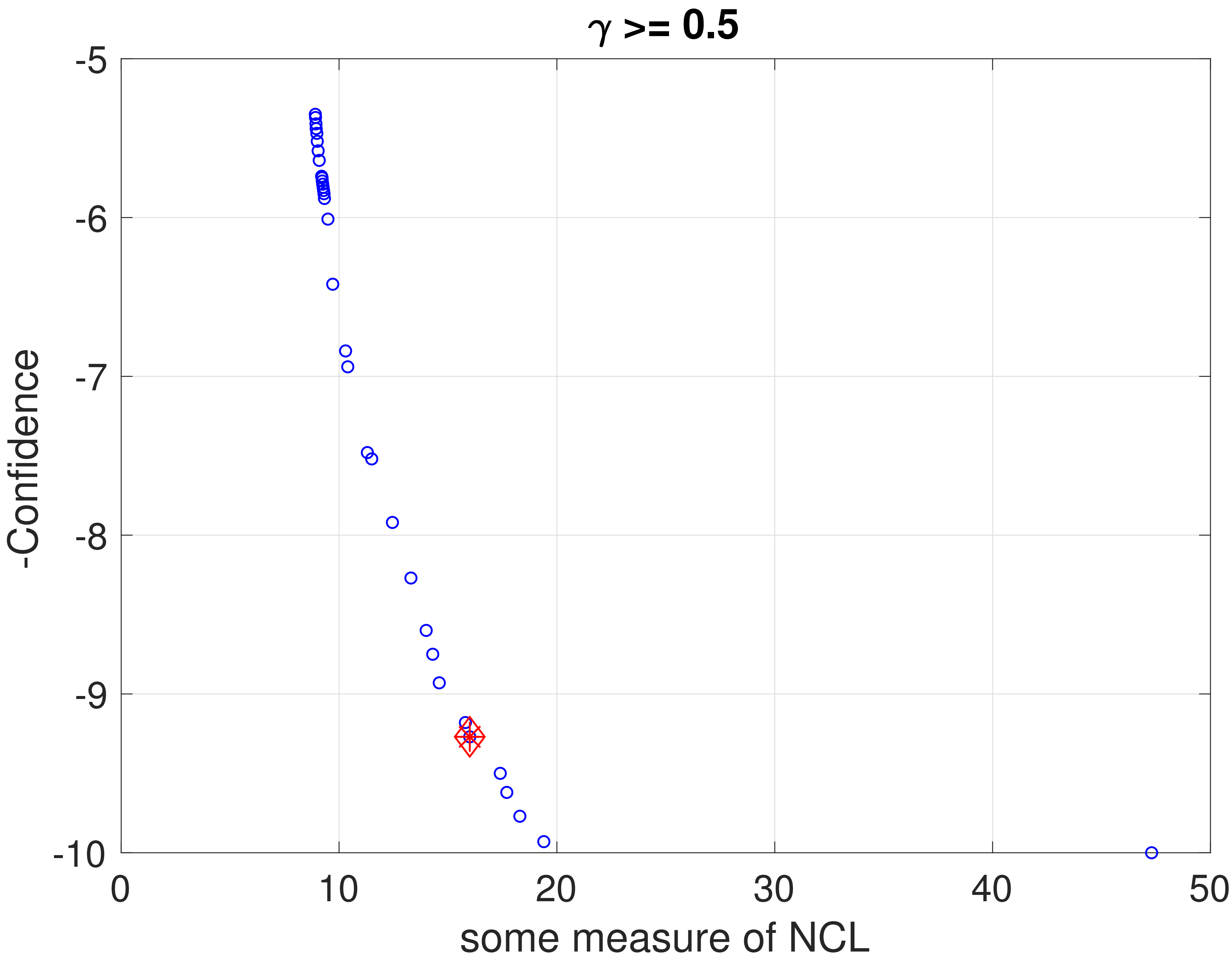
$$
\text{(MOPs)} \begin{cases} \displaystyle \min_{x,v,u} \quad \max\left\{ -w \int_0^{t_f} \gamma(x(t),t)\,dt,\ (1-w)\int_0^{t_f} \tilde{\varphi}(x(t),t)\,dt \right\} \\[2ex] \quad \text{s.t.} \quad \dot{x}(t) = v(t)\,, \qquad (x(0) = x_0)\,, \\ \qquad\qquad \dot{v}(t) = u(t)\,, \qquad (v(0) = v_0)\,, \\ \qquad\qquad\qquad\qquad\qquad v_{\min} \le v(t) \le v_{\max}\,, \end{cases}
$$

where $0 \le w \le 1$. The scalarization used here is referred to as Chebyschev scalarization in the literature.

The Pareto Front

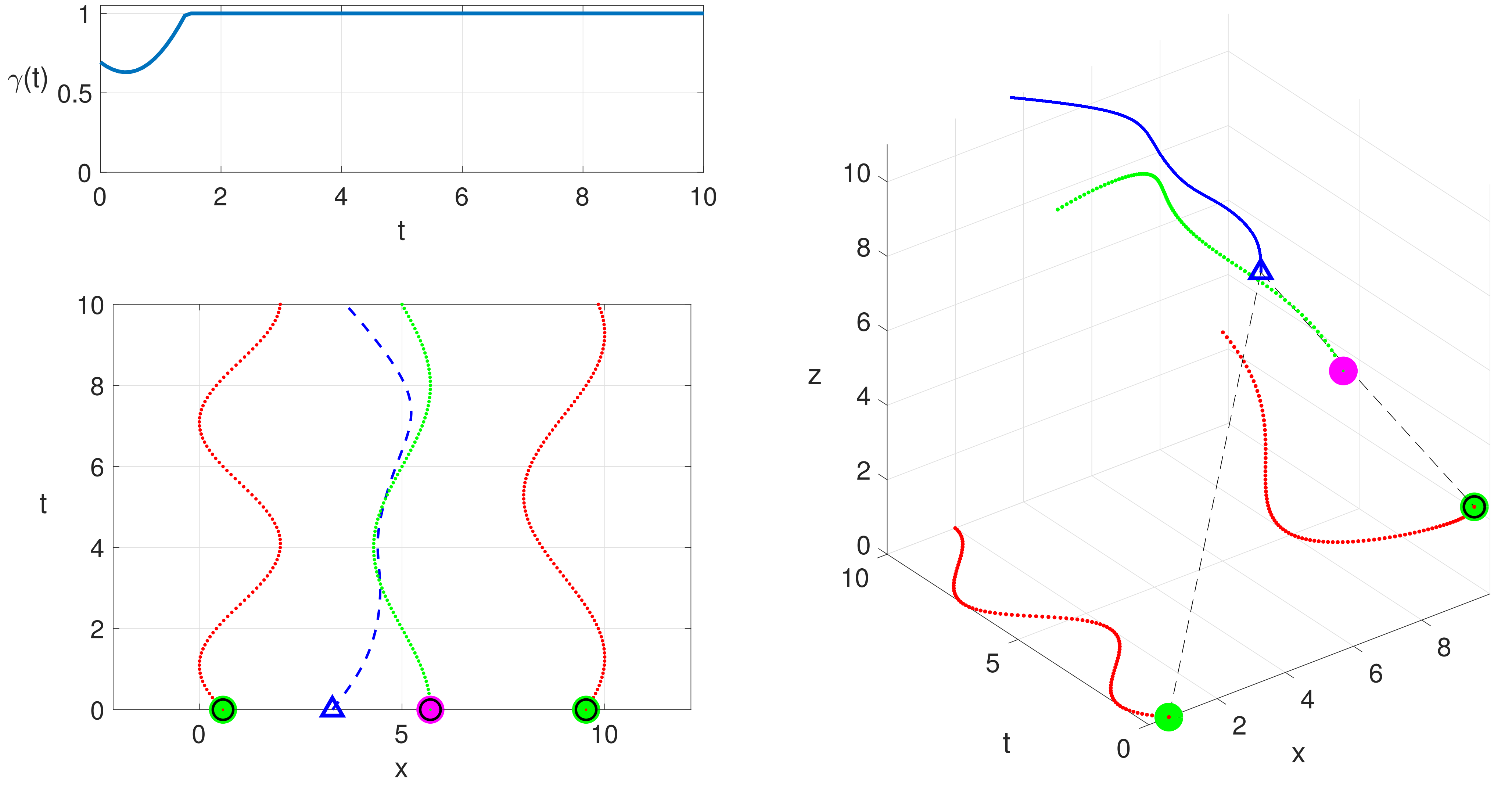


The Pareto Front



Decision Maker's Solution

Partial confidence (View the movie!)



Summary

- We have proposed and implemented a function as a measure of network connection level in the optimal control of a UAV in a dynamic network.
- We have also introduced a confidence level function for connectivity and implemented it.
- We formulated a multi-objective optimal control of a UAV over dynamic networks.
- We proposed and implemented numerical methods to solve these three separate problems.
- Plenty more to do for the future: More nodes (ground and airborne), uncertainty appearing in further dimensions, more objective functionals, non-smooth numerical techniques, etc.