A comparison of nonlinear, second order cone and linear programming formulations for the optimal power flow problem

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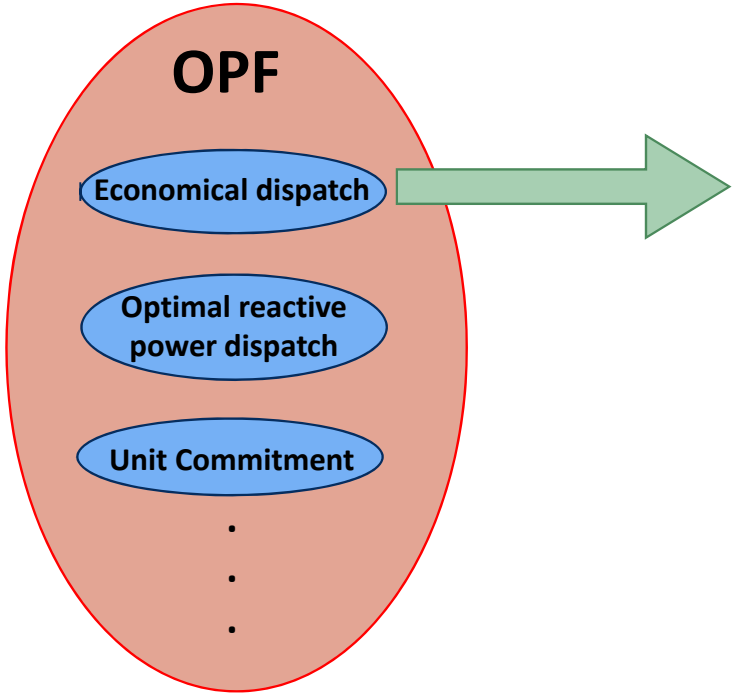
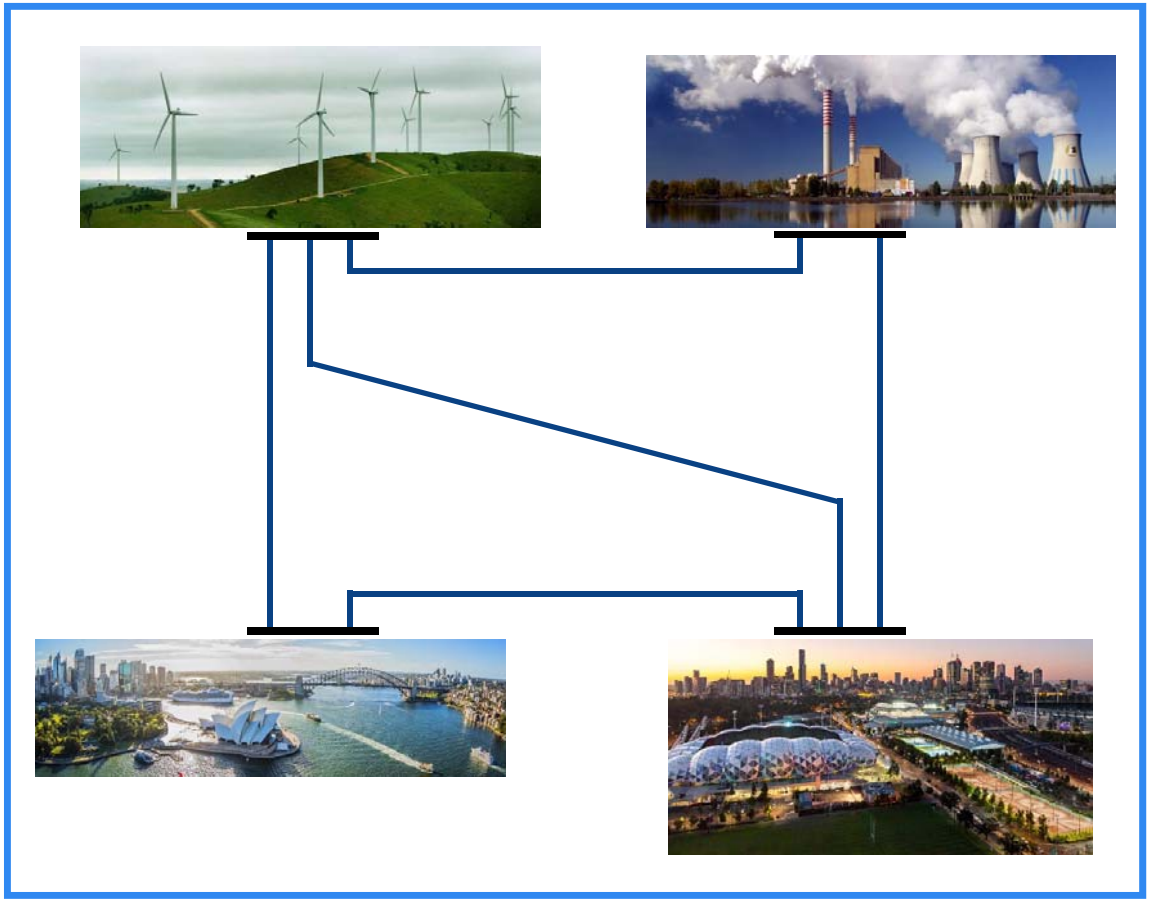
Optimal Power Flow

What is the Optimal Power Flow?

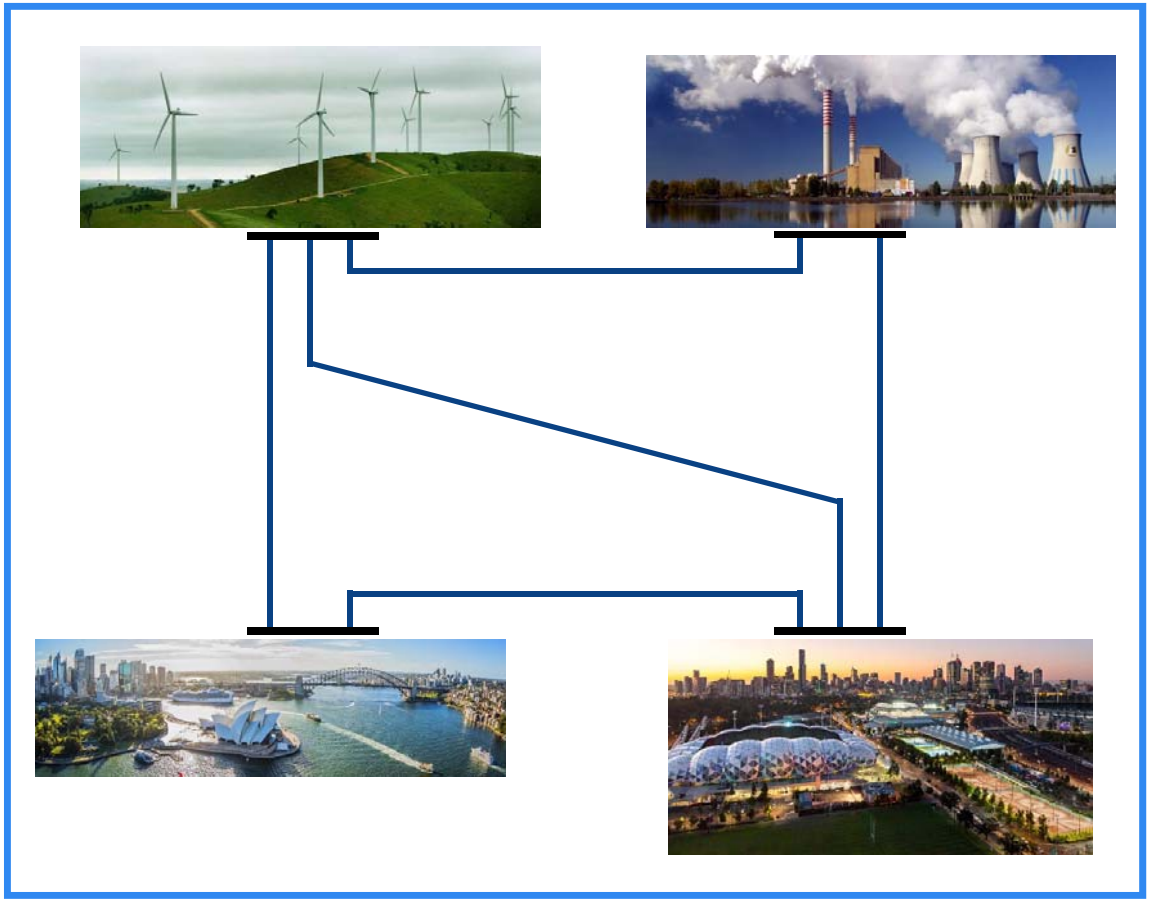
The OPF defines a set of electrical power systems problems that are subjected to power flow constraints and other operational constraints.





What is the importance of the economical dispatch?

- Exist many ways to dispatch the generation.
- Some could be very expensive.
- The ED determine the lowest cost generation dispatch.



Why is it important to improve the OPF mathematical formulation?

[1] 257.000.000 **MWh**

X

[2] 104 **AUD$/MWh**

=

AUD$ 26728 Millions

5%

AUD$ 1300 Millions

[1] 'Australian Energy Update', Department of the Environment and Energy, Australia, 2017.
[2] S. Letts, 'Power prices are 'off the chart' and there's no relief in sight', *ABC News*, July 7, 2017. [Online]. Available: http://www.abc.net.au/news/2017-07-07/power-prices-off-the-chart/8687480. [Accessed June 1, 2018].

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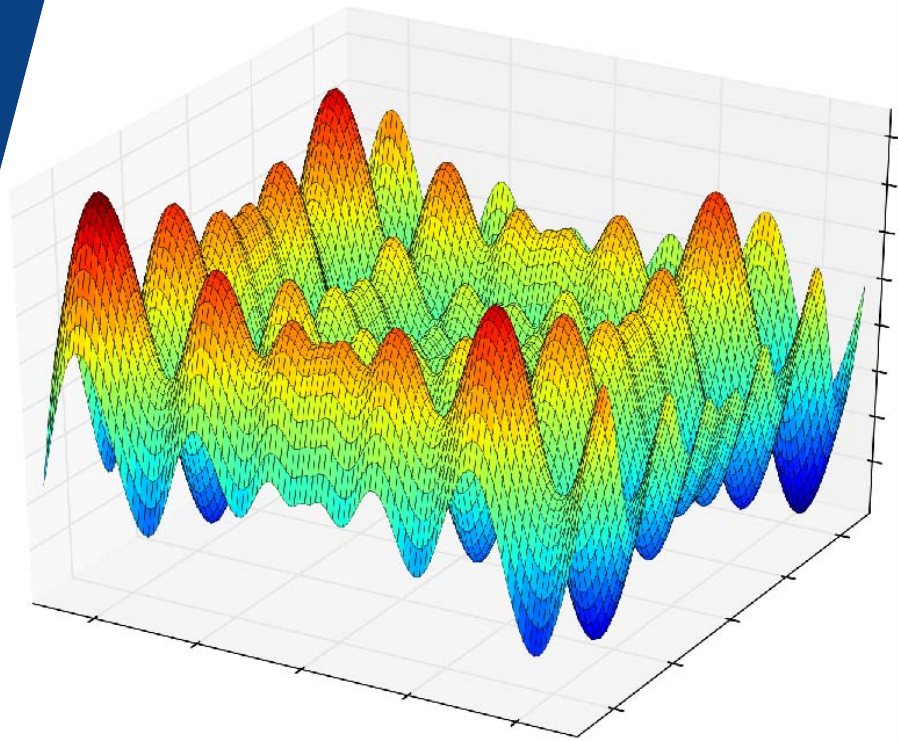
OPF formulations



Polar Power and Voltage - Nonlinear

Objective function.

$$\min \sum_{n\in\Omega_g} \left(C_n^{g2}(p_n^g)^2 + C_n^{g1} p_n^g + C_n^{g0}\right)$$

Power balance.

$$p_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} P_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} P_{(l,n,m)}^{to} + g_n^{sh} v_n^2 + P_n^d$$

$$q_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} Q_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} Q_{(l,n,m)}^{to} - b_n^{sh} v_n^2 + Q_n^d$$

Power flow equations.

$$P_{(l,n,m)}^{from} = a_{(l,n,m)}^2 v_n^2 g_{(l,n,m)} - a_{(l,n,m)} v_n v_m \left(g_{(l,n,m)} \cos(\theta_n - \theta_m + \varphi_{(l,n,m)}) + b_{(l,n,m)} \sin(\theta_n - \theta_m + \varphi_{(l,n,m)})\right)$$

$$Q_{(l,n,m)}^{from} = -a_{(l,n,m)} v_n v_m \left(g_{(l,n,m)} \sin(\theta_n - \theta_m + \varphi_{(l,n,m)}) - b_{(l,n,m)} \cos(\theta_n - \theta_m + \varphi_{(l,n,m)})\right) - a_{(l,n,m)}^2 v_n^2 (b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

$$P_{(l,n,m)}^{to} = v_m^2 g_{(l,n,m)} - a_{(l,n,m)} v_n v_m \left(g_{(l,n,m)} \cos(\theta_n - \theta_m + \varphi_{(l,n,m)}) - b_{(l,n,m)} \sin(\theta_n - \theta_m + \varphi_{(l,n,m)})\right)$$

$$Q_{(l,n,m)}^{to} = a_{(l,n,m)} v_n v_m \left(g_{(l,n,m)} \sin(\theta_n - \theta_m + \varphi_{(l,n,m)}) + b_{(l,n,m)} \cos(\theta_n - \theta_m + \varphi_{(l,n,m)})\right) - v_m^2 (b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

Bounds.

$$P_n^{min} \le p_n^g \le P_n^{max} \qquad (P_{(l,n,m)}^{to})^2 + (Q_{(l,n,m)}^{to})^2 \le (S_{(l,n,m)}^{max})^2$$

$$Q_n^{min} \le q_n^g \le Q_n^{max} \qquad (P_{(l,n,m)}^{from})^2 + (Q_{(l,n,m)}^{from})^2 \le (S_{(l,n,m)}^{max})^2 \qquad v_n^{min} \le v_n \le v_n^{max}$$

Non Convex Multimodal

Convex quadratic constraints.

Squared variables. Equality constraints.

Trigonometric functions. Product of variables. Equality constraints.

Convex constraints.

Polar Power and Voltage – Second Order Cone

Power balance.

$$p_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} P_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} P_{(l,n,m)}^{to} + g_n^{sh}\sqrt{2}u_n + P_n^d$$

$$q_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} Q_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} Q_{(l,n,m)}^{to} - b_n^{sh}\sqrt{2}u_n + Q_n^d$$

Power flow equations.

$$P_{(l,n,m)}^{from} = a_{(l,n,m)}^2\sqrt{2}u_n g_{(l,n,m)} - a_{(l,n,m)}\left(g_{(l,n,m)}\beta_{(l,n,m)} + b_{(l,n,m)}\lambda_{(l,n,m)}\right)$$

$$Q_{(l,n,m)}^{from} = -a_{(l,n,m)}\left(g_{(l,n,m)}\lambda_{(l,n,m)} - b_{(l,n,m)}\beta_{(l,n,m)}\right) - a_{(l,n,m)}^2\sqrt{2}u_n(b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

$$P_{(l,n,m)}^{to} = \sqrt{2}u_m g_{(l,n,m)} - a_{(l,n,m)}\left(g_{(l,n,m)}\beta_{(l,n,m)}) - b_{(l,n,m)}\lambda_{(l,n,m)}\right)$$

$$Q_{(l,n,m)}^{to} = a_{(l,n,m)}\left(g_{(l,n,m)}\lambda_{(l,n,m)} + b_{(l,n,m)}\beta_{(l,n,m)}\right) - \sqrt{2}u_m(b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

Bounds.

$$P_n^{min} \le p_n^g \le P_n^{max} \qquad (P_{(l,n,m)}^{to})^2 + (Q_{(l,n,m)}^{to})^2 \le (S_{(l,n,m)}^{max})^2 \qquad \frac{\left(v_n^{min}\right)^2}{\sqrt{2}} \le u_n \le \frac{\left(v_n^{max}\right)^2}{\sqrt{2}}$$

$$Q_n^{min} \le q_n^g \le Q_n^{max} \qquad (P_{(l,n,m)}^{from})^2 + (Q_{(l,n,m)}^{from})^2 \le (S_{(l,n,m)}^{max})^2$$

Additional Constraints

$$2u_n u_m \ge \lambda_{(l,n,m)}^2 + \beta_{(l,n,m)}^2$$

$$\lambda_{(l,n,m)} = \theta_n - \theta_m + \varphi_{(l,n,m)}$$

Substitutions

$$u_n = \frac{v_n^2}{\sqrt{2}}$$

$$\beta_{(l,n,m)} = v_n v_m \cos(\theta_n - \theta_m + \varphi_{(l,n,m)})$$

$$\lambda_{(l,n,m)} = v_n v_m \sin(\theta_n - \theta_m + \varphi_{(l,n,m)})$$

$$2u_n u_m = \lambda_{(l,n,m)}^2 + \beta_{(l,n,m)}^2$$

Original Constraints

Approximations

$$v_n \approx 1$$

$$\sin(\theta_n - \theta_m + \varphi_{(l,n,m)}) \approx \theta_n - \theta_m + \varphi_{(l,n,m)}$$

Polar Power and Voltage – Second Order Cone

Power balance.

$$p_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} P_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} P_{(l,n,m)}^{to} + g_n^{sh}\sqrt{2}u_n + P_n^d$$

$$q_n^g = \sum_{\substack{(l,n,m)\in\Omega_l \\ n<m}} Q_{(l,n,m)}^{from} + \sum_{\substack{(l,n,m)\in\Omega_l \\ n>m}} Q_{(l,n,m)}^{to} - b_n^{sh}\sqrt{2}u_n + Q_n^d$$

Power flow equations.

$$P_{(l,n,m)}^{from} = a_{(l,n,m)}^2\sqrt{2}u_n g_{(l,n,m)} - a_{(l,n,m)}\left(g_{(l,n,m)}\beta_{(l,n,m)} + b_{(l,n,m)}\lambda_{(l,n,m)}\right)$$

$$Q_{(l,n,m)}^{from} = -a_{(l,n,m)}\left(g_{(l,n,m)}\lambda_{(l,n,m)} - b_{(l,n,m)}\beta_{(l,n,m)}\right) - a_{(l,n,m)}^2\sqrt{2}u_n(b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

$$P_{(l,n,m)}^{to} = \sqrt{2}u_m g_{(l,n,m)} - a_{(l,n,m)}\left(g_{(l,n,m)}\beta_{(l,n,m)}) - b_{(l,n,m)}\lambda_{(l,n,m)}\right)$$

$$Q_{(l,n,m)}^{to} = a_{(l,n,m)}\left(g_{(l,n,m)}\lambda_{(l,n,m)} + b_{(l,n,m)}\beta_{(l,n,m)}\right) - \sqrt{2}u_m(b_{(l,n,m)}^{sh} + b_{(l,n,m)})$$

Bounds.

$$P_n^{min} \le p_n^g \le P_n^{max} \qquad (P_{(l,n,m)}^{to})^2 + (Q_{(l,n,m)}^{to})^2 \le (S_{(l,n,m)}^{max})^2 \qquad \frac{\left(v_n^{min}\right)^2}{\sqrt{2}} \le u_n \le \frac{\left(v_n^{max}\right)^2}{\sqrt{2}}$$

$$Q_n^{min} \le q_n^g \le Q_n^{max} \qquad (P_{(l,n,m)}^{from})^2 + (Q_{(l,n,m)}^{from})^2 \le (S_{(l,n,m)}^{max})^2$$

Additional Constraints

$$2u_n u_m \ge \lambda_{(l,n,m)}^2 + \beta_{(l,n,m)}^2$$

$$\lambda_{(l,n,m)} = \theta_n - \theta_m + \varphi_{(l,n,m)}$$

Convex Global Optima

Linear Constraints.

Linear Constraints.
Conic Constraints.

DC Power Flow- Linear

Objective function.

$$L\left(\min \sum_{n\in\Omega_g}\left(C_n^{g2}(p_n^g)^2 + C_n^{g1}p_n^g + C_n^{g0}\right)\right)$$

Power balance.

$$p_n^g = \sum_{(l,n,m)\in\Omega_l} P_{(l,n,m)} - \sum_{(l,m,n)\in\Omega_l} P_{(l,m,n)} + P_n^d$$

Power flow equations.

$$P_{(l,n,m)} = b_{(l,n,m)}\left(\theta_n - \theta_m + \varphi_{(l,n,m)}\right)$$

Bounds.

$$P_n^{min} \le p_n^g \le P_n^{max}$$

$$-P_{(l,n,m)}^{max} \le P_{(l,n,m)} \le P_{(l,n,m)}^{max}$$

Piecewise linear function

Approximations

$$v_n \approx 1$$

$$\sin(\theta_n - \theta_m + \varphi_{(l,n,m)}) \approx \theta_n - \theta_m + \varphi_{(l,n,m)}$$

$$\cos(\theta_n - \theta_m + \varphi_{(l,n,m)}) \approx 1$$

$$r_{(l,n,m)} \square \; x_{(l,n,m)}$$

$$\cancel{\varphi}_{(l,n,m)}$$

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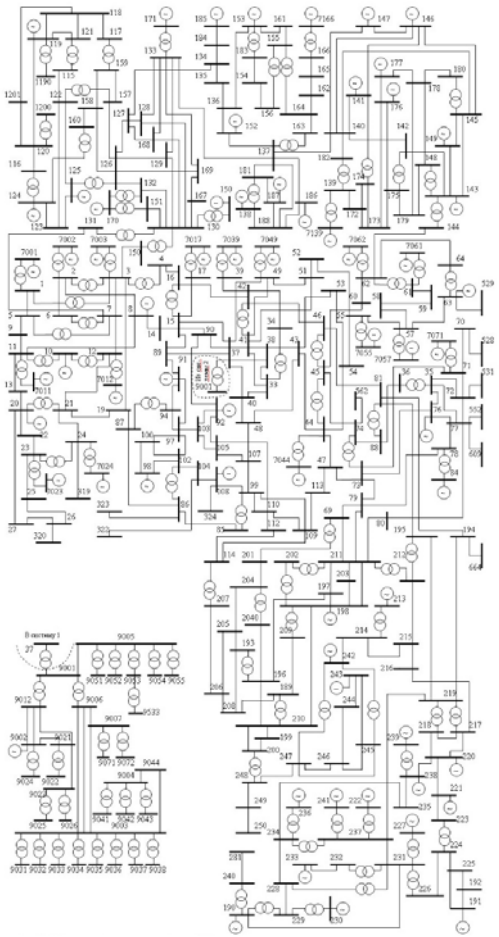
Simulations and Results

Test system and optimisation tools



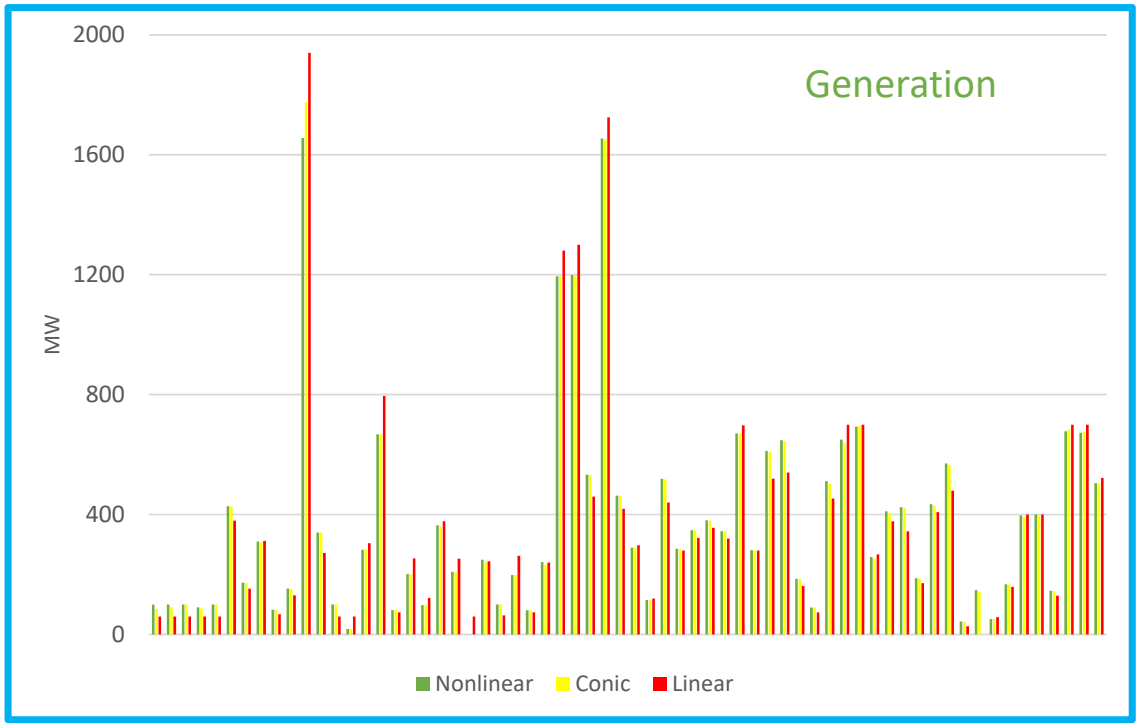
IEEE 300

The mathematical formulations were implemented in AMPL.

The nonlinear formulation was solved using KNITRO.

The SOC and linear formulations were solved with CPLEX.

Active Power Generation and Cost

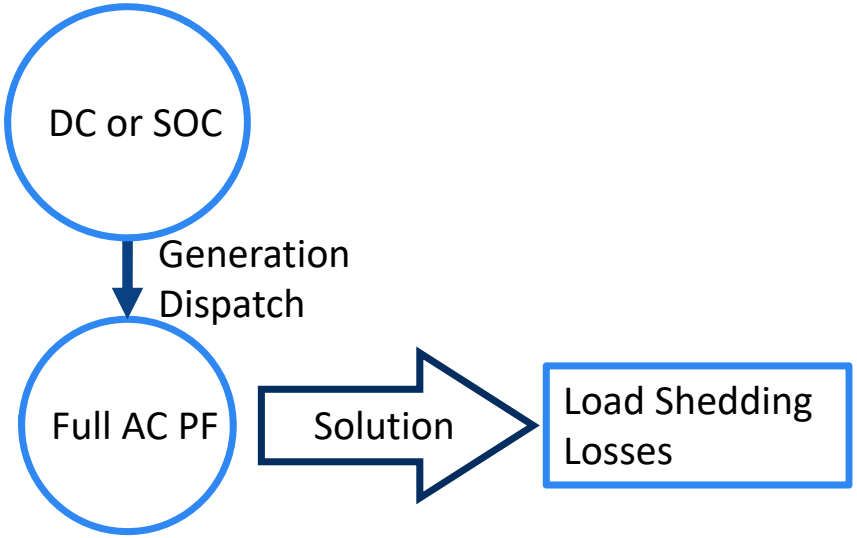


	Cost US$/hr
Nonlinear	722670,42
Conic	721707,75
Linear	709066,95

	Generation (MW)
Nonlinear	23690,15
Conic	23681,31
Linear	23391,01

Active Power Generation and Cost

Testing the feasibility of the solutions



	Cost US$/hr
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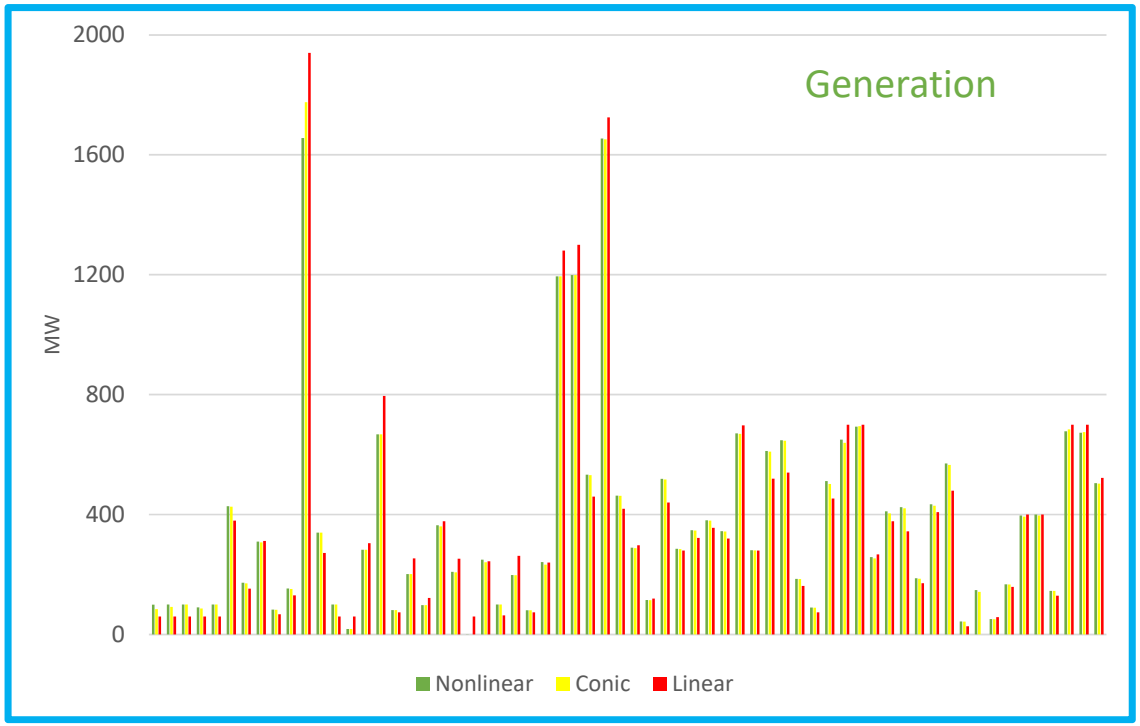
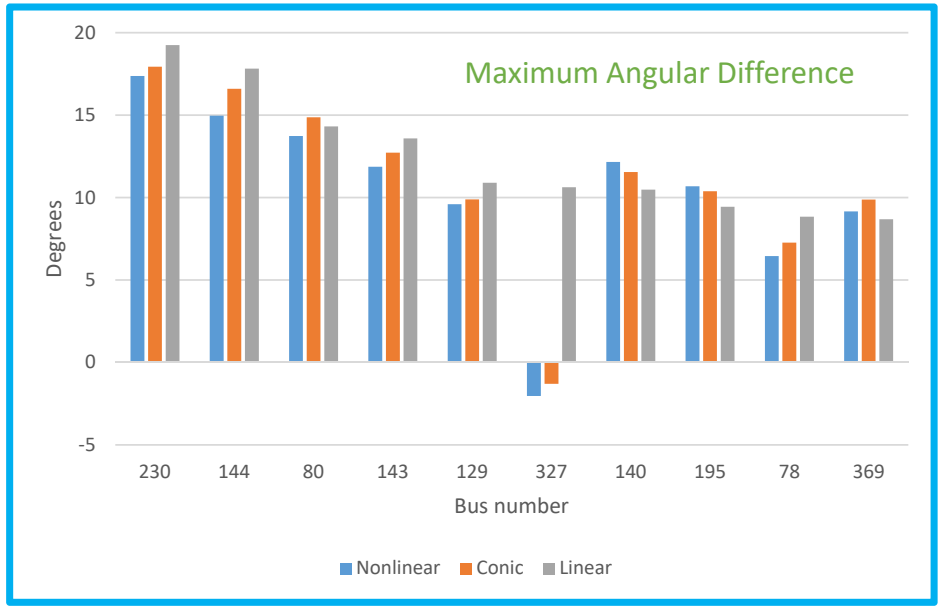
Active Power Generation and Cost

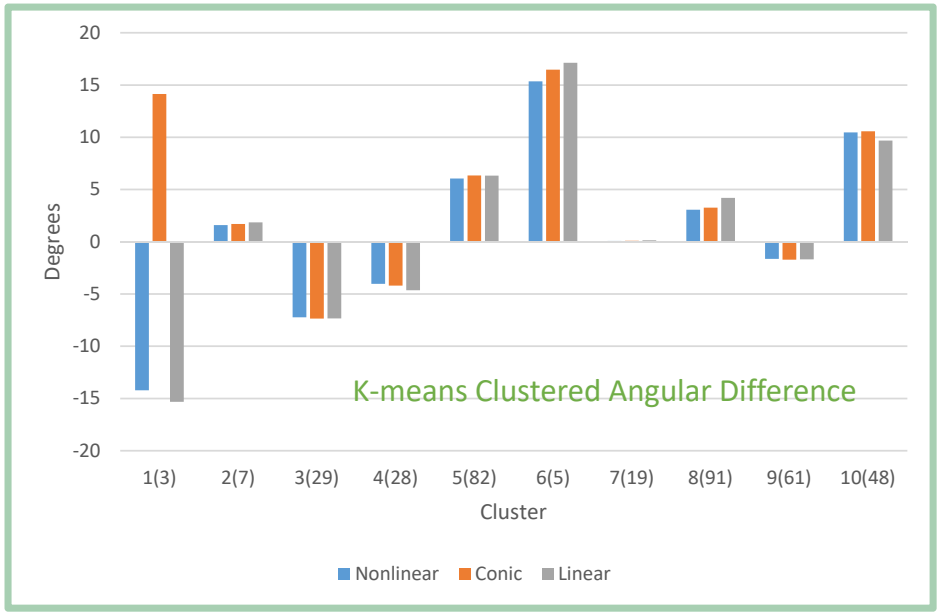


	Load Shedding (MW)
SOC	37,32
Linear	1005,76

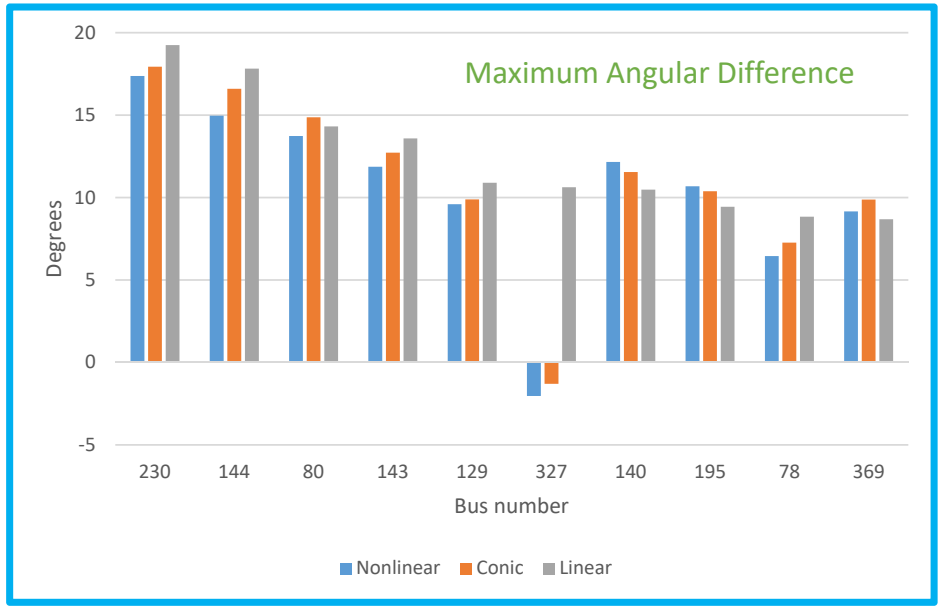
Infeasible!

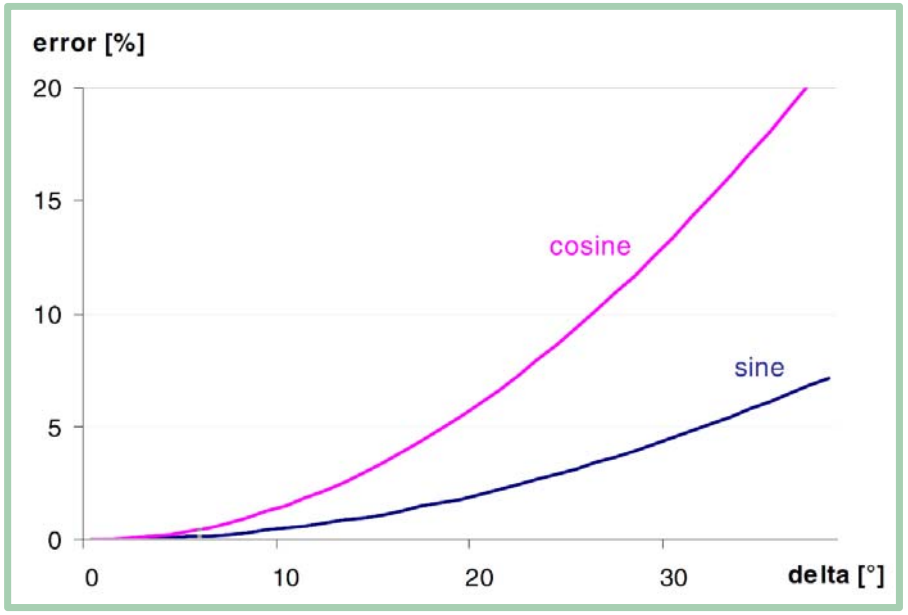
Voltage Angle Difference



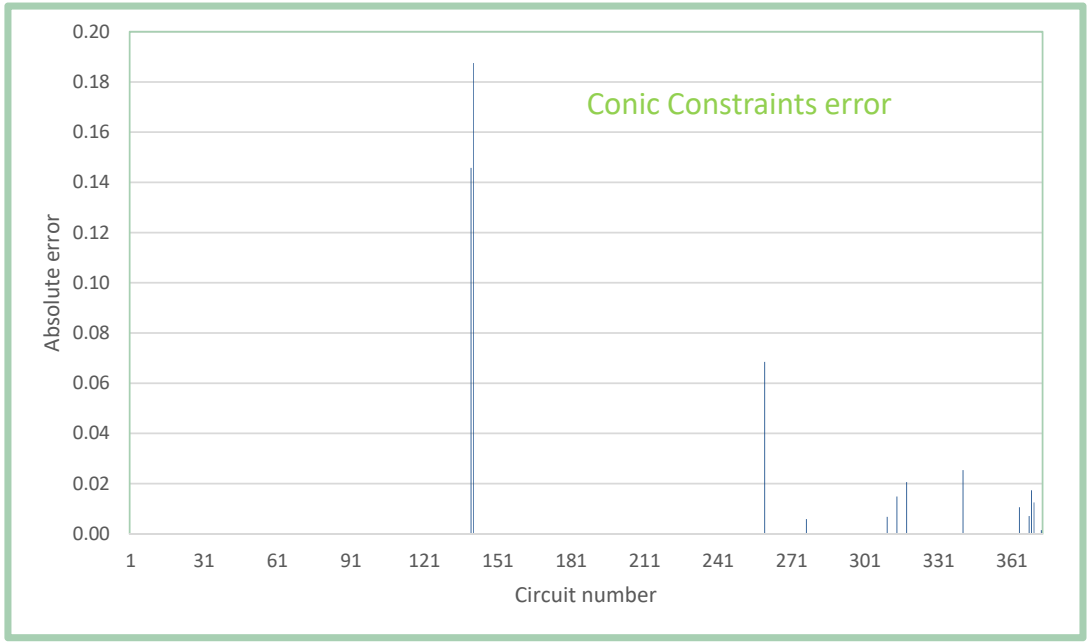


Voltage Angle Difference





Conic Constraints Approximation Error



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Conclusions

Preliminary Conclusions

In this preliminary research **the solution obtained with the DC OPF and the second order cone approximations are infeasible for the full AC OPF representing the IEEE-300** test system with power limit for the transmission lines and transformers. However, further studies need to be done in order to determine if these results are particular for this test system or if a fundamental problem has been detected.

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Thank you

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