Generating mixed integer programming instances with challenging properties

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AMSI Optimise
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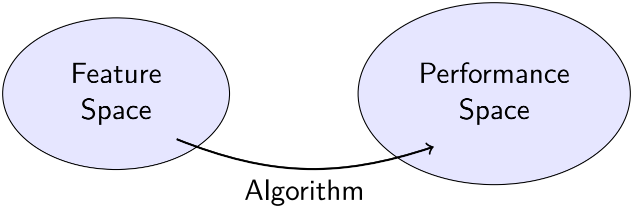
Outline



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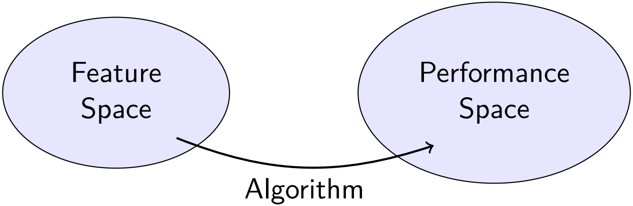


Give an overview of...

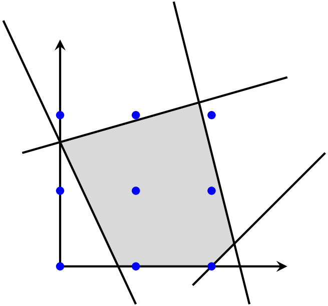
- MIP solvers (in particular branching strategies)
- Generating new instances using evolutionary algorithms
- Characteristics of the generated instances

MIP/Polyhedra

Mixed Integer Program

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

some or all x_i integral



MIP Solvers

Mixed Integer Program

$$
\begin{aligned}
&\text{maximize} && c^T x \\
&\text{subject to} && Ax \leq b \\
& && x \geq 0 \\
&\text{some or all } x_i \text{ integral}
\end{aligned}
$$

Many interacting components make up a MIP solver:

- Presolvers
- Cutting planes
- Primal heuristics
- Parallelisation
- LP solvers
- Branching rules
- Node selection rules
- Domain propagation

MIP Solvers

Mixed Integer Program

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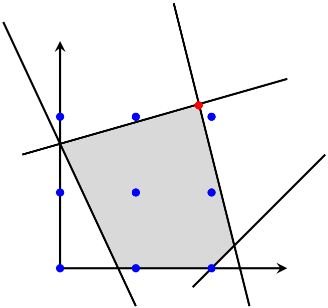
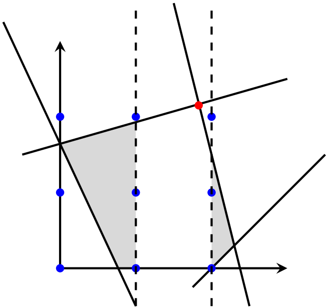
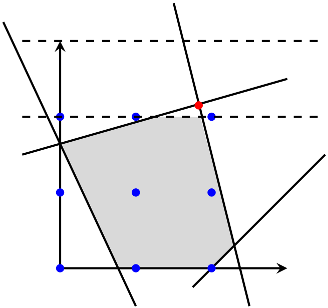
Branching Strategies



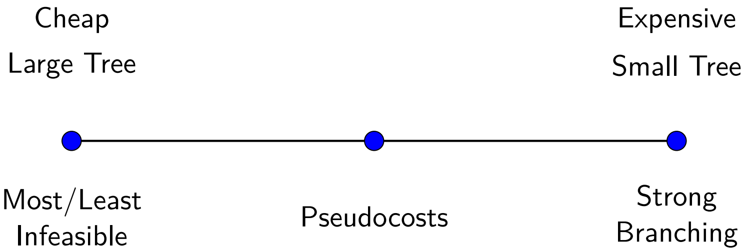
Branching Strategies



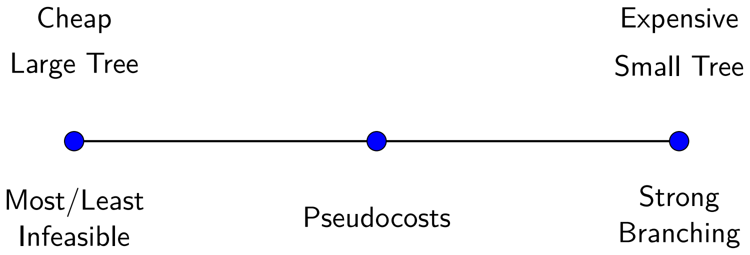
Branching Strategies



Branching Strategies



Branching Strategies



To conduct a branching study . . .

- Disable cuts beyond the root node
- Provide the optimal solution

(Linderoth and Savelsbergh, 1999)

Evolving Instances

Genetic algorithms

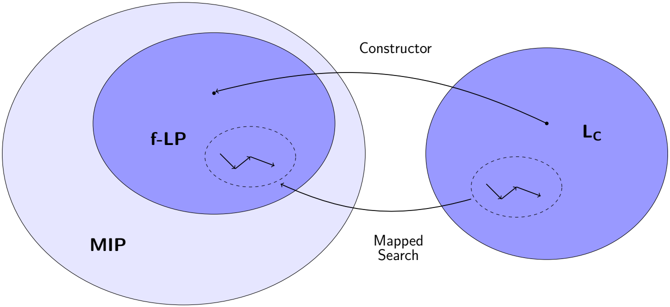
- Population based metaheuristic with fitness selection
- Crossover is the key search operator

Previous applications to instance generation

- Finding worst-case bounds for sorting algorithm performance (Cotta and Moscato, 2003).
- Exposing strengths and weaknesses of heuristics (van Hemert, 2006; Langdon and Poli, 2007).

Evolving Instances

How can we narrow the search space?



Restrict search to feasible, bounded problems.

Evolving Instances

Dual programs

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax + s = b \\ & x, s \geq 0 \end{aligned} \iff \begin{aligned} \min \quad & b^T y \\ \text{s.t.} \quad & A^T y - r = c \\ & y, r \geq 0 \end{aligned}$$

Optimality conditions

(x, s) and (y, r) feasible to primal and dual
$x_i r_i = 0 \quad \forall\, i$
$y_j s_j = 0 \quad \forall\, j$

Construction

$$\begin{aligned} b &= Ax + s \\ c &= A^T y - r \end{aligned}$$

Evolving Instances

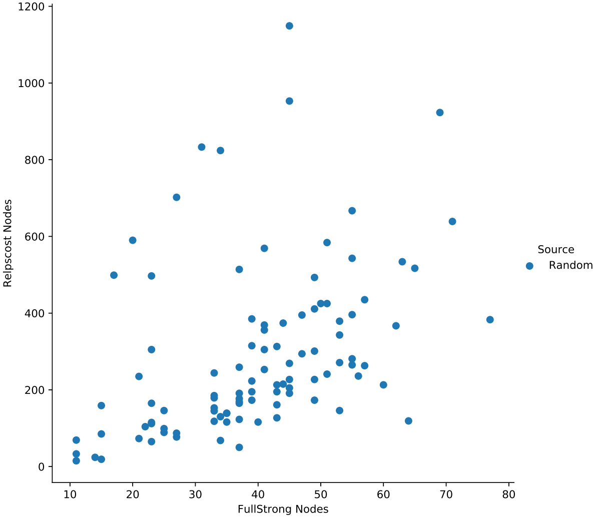
- Uniform row crossover:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ a_{31} & \dots & a_{3n} \end{bmatrix} \quad \begin{bmatrix} b_{11} & \dots & b_{1n} \\ b_{21} & \dots & b_{2n} \\ b_{31} & \dots & b_{3n} \end{bmatrix}$$

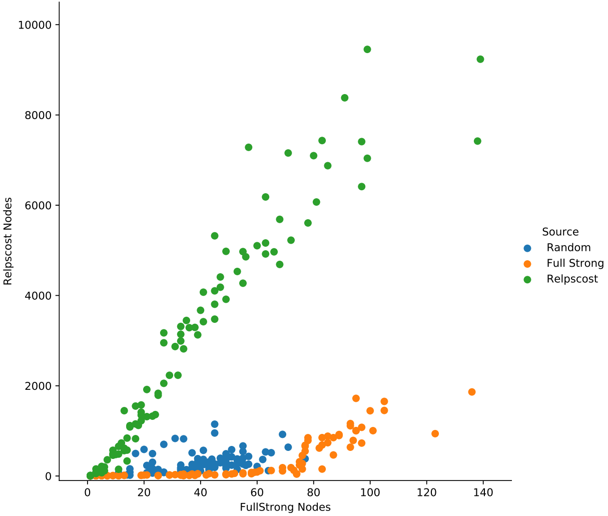
$$\begin{bmatrix} a_{21} & \dots & a_{2n} \\ b_{21} & \dots & b_{2n} \\ a_{31} & \dots & a_{3n} \end{bmatrix} \quad \begin{bmatrix} a_{11} & \dots & a_{1n} \\ b_{31} & \dots & b_{3n} \\ b_{11} & \dots & b_{1n} \end{bmatrix}$$

- Multi-objective NSGA-II algorithm (Deb et al., 2002)
- Small instances - 50 integer variables and 50 constraints
- *fullstrong* and *relpscost* branching strategies in SCIP 6.0.0

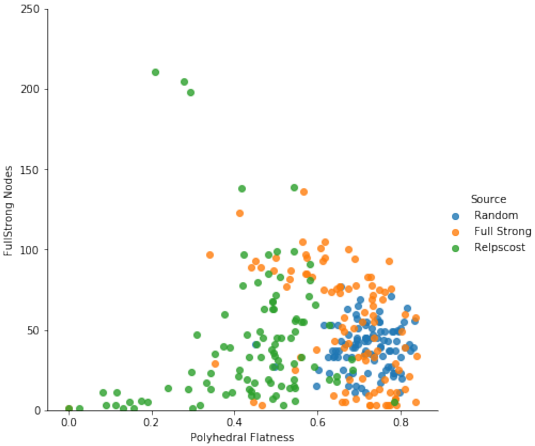
Performance Discriminating Instances



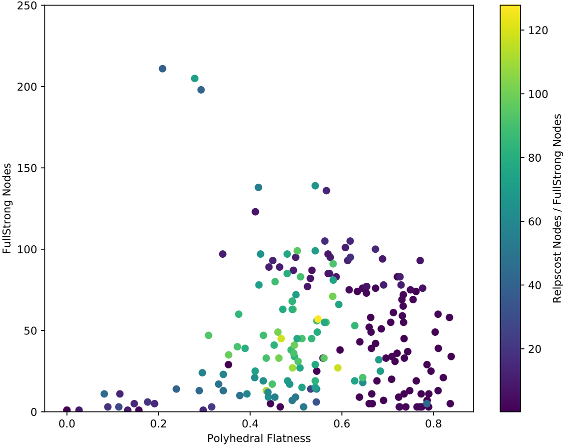
Performance Discriminating Instances



Properties of Generated Instances



Properties of Generated Instances



Takeaways

- Evolutionary generation allows us to search performance space by manipulating instances.
- Small instances can be generated which expose performance differences in fundamental components.
- Branching is just one facet of MIP solver performance - there's more of the performance space to explore.

References

Cotta, C. and Moscato, P. (2003). A mixed evolutionary-statistical analysis of an algorithm's complexity. *Applied Mathematics Letters*, 16(1):41–47.

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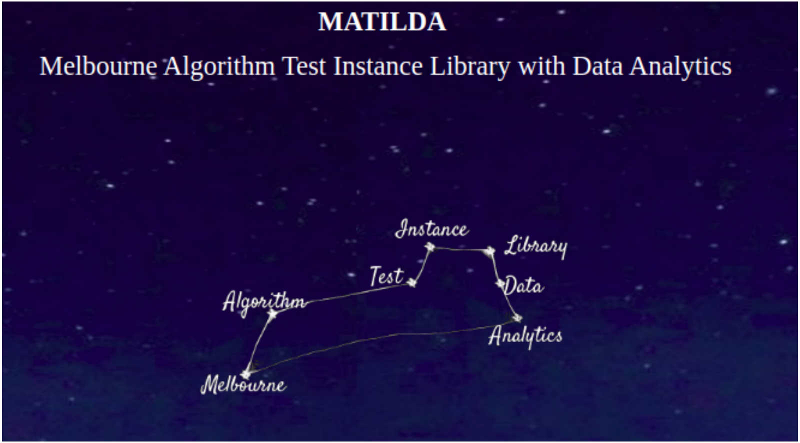
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Systems Specialist & Web Developer

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Footprint Analysis

GRAPH COLORING PROBLEM

INSTANCE SPACE 2D COORDINATES = PROJECTION MATRIX X FEATURES VECTOR

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 0.559 & -0.702 \\ 0.614 & -0.007 \\ 0.557 & 0.712 \end{bmatrix}^T \begin{bmatrix} Density \\ AlgConnectivity \\ Energy \end{bmatrix}$$

Select a graph to visualize:

SVM Selection