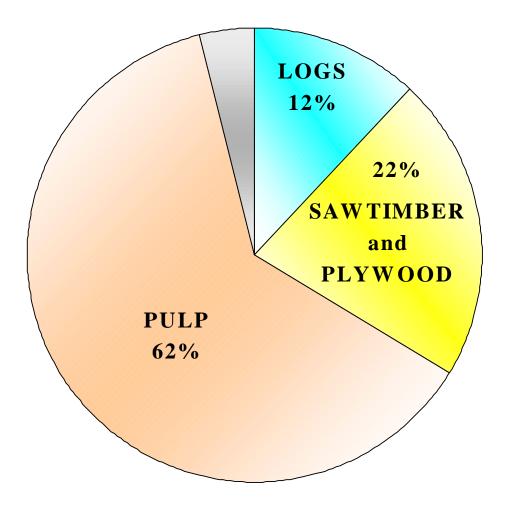
# Optimization of Forest Industry Operations

AMSI 2019, PERTH RAFAEL EPSTEIN UNIVERSIDAD DE CHILE

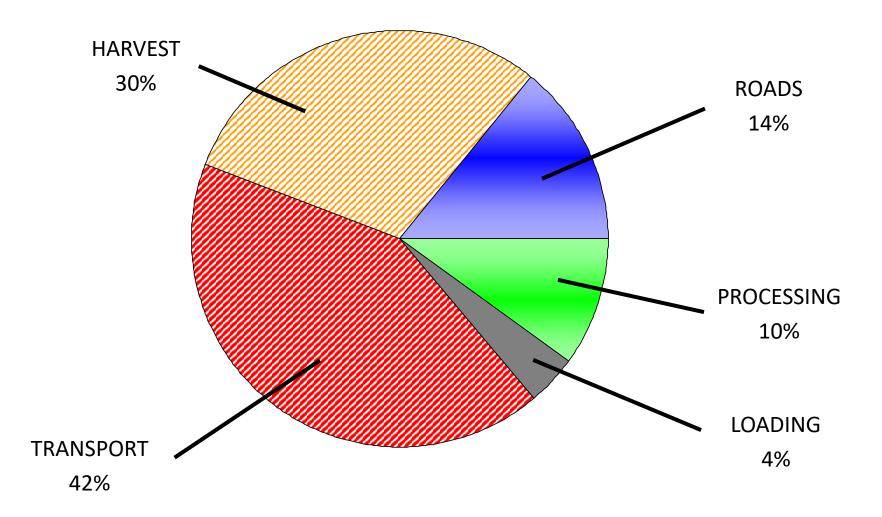
#### Forest Industry

- Multiple processes: where to focus?
- Highly competitive, hard work.
- Driven by cost:
  - Efficiency is mandatory.

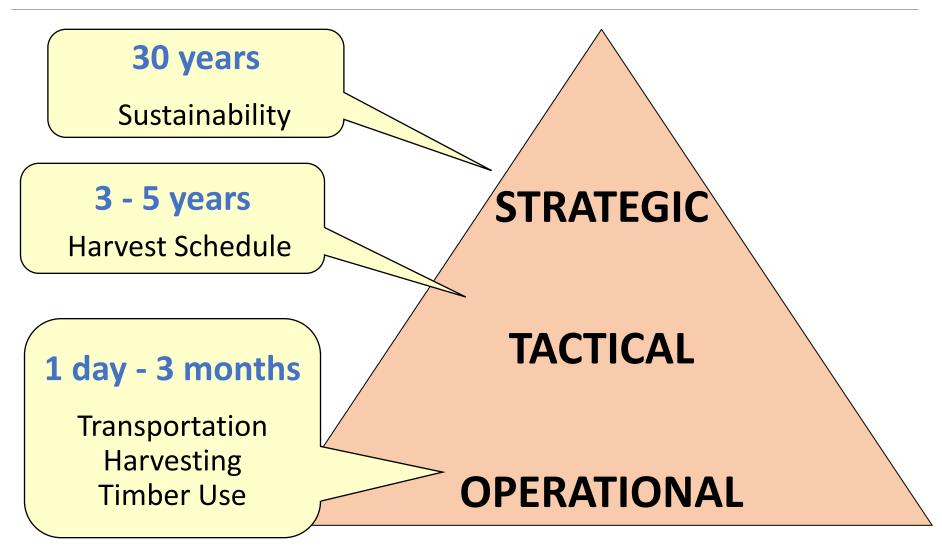
#### Forest Industry



#### Forest Industry



#### **Decision Levels**



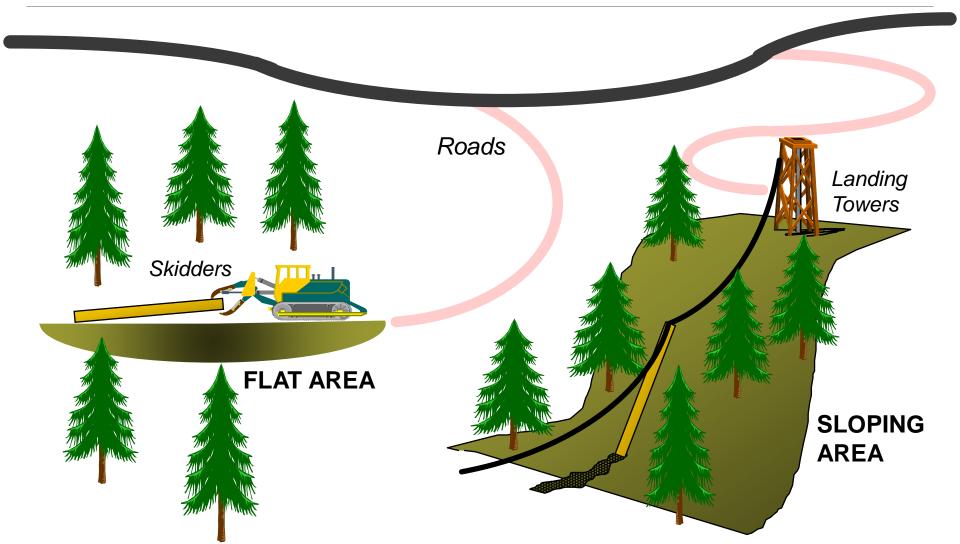
#### Focused on Main Problems

OPERATIONAL PROBLEMS

Transportation Harvesting

**Timber Use** 

- Main decisions:
  - Where to locate the harvesting machinery.
  - Which areas to each machine.
  - The road network needed for extraction.

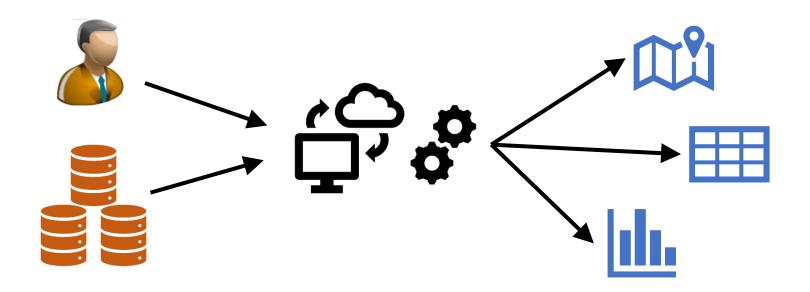




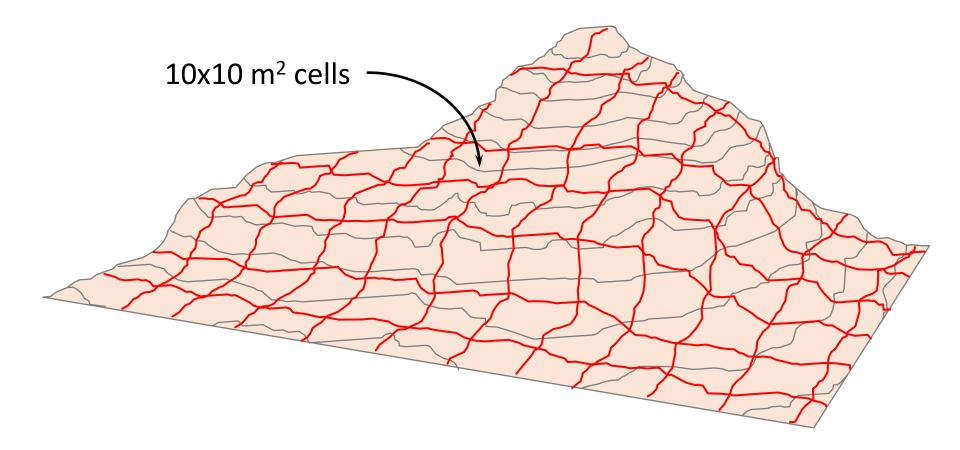
- The objective: minimize the total cost (road + harvesting).
- Main constrains:
  - Technical constrains for the harvesting machinery:
    - Maximum forwarding slope.
    - Maximum side slope.
  - Technical constrains for the roads:
    - Maximum slope.
    - Turning angle for the forest trucks.
  - Environmental constrains:
    - Protected areas.
    - Earth movement.

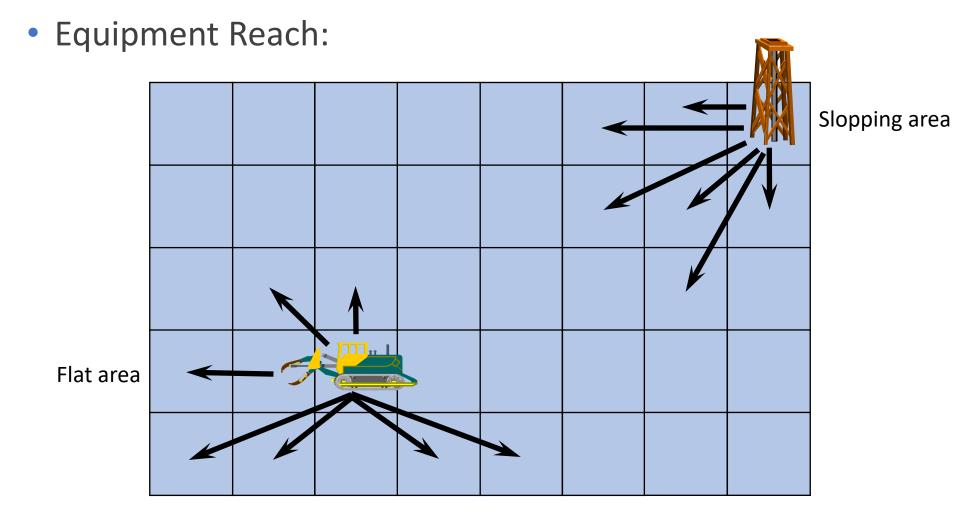
- Traditional approach:
  - An iterative process between a well experienced planning engineer and in-field analysis.
- Main drawbacks:
  - The iteration process is slow and inefficient.
  - Such experience is hard to obtain.
  - The results can be easily biased from said experience.
  - Given its complexity, it is impossible for a human to incorporate all the variables in the analysis.

- Analytical approach:
  - With the use of geographical data, we solve the problem with an optimization model.

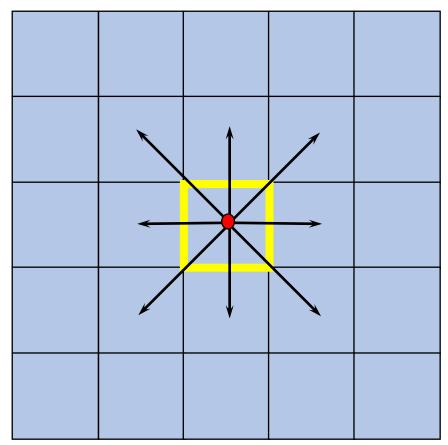


- Area divided in 10x10 m<sup>2</sup> cells.
- GIS provides topological information.
- Uncapacitated Network Design Model:
  - Route a single product from different origins to destinies.
  - Fixed-charge for using an arc.
  - Per unit flow costs on each arc.
  - NP-hard (special case: Steiner Tree).
- Solved heuristically.

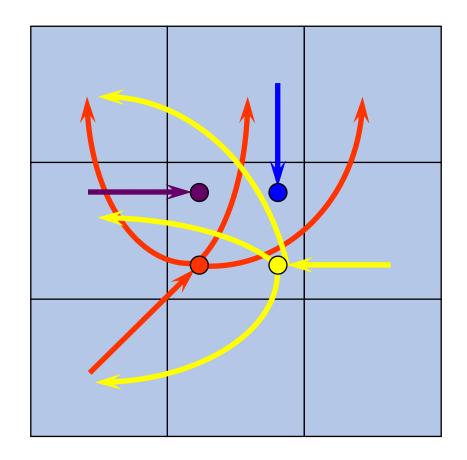


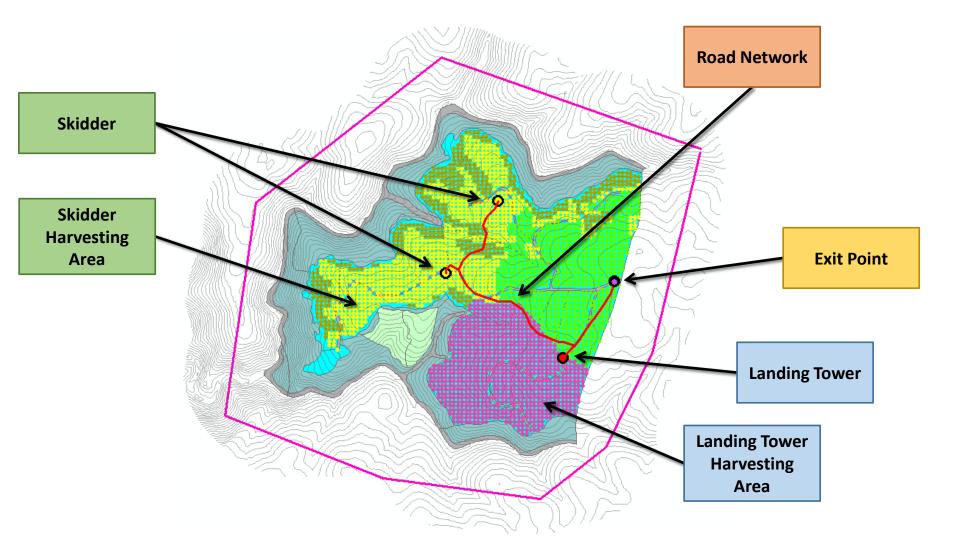


• Road Segments:

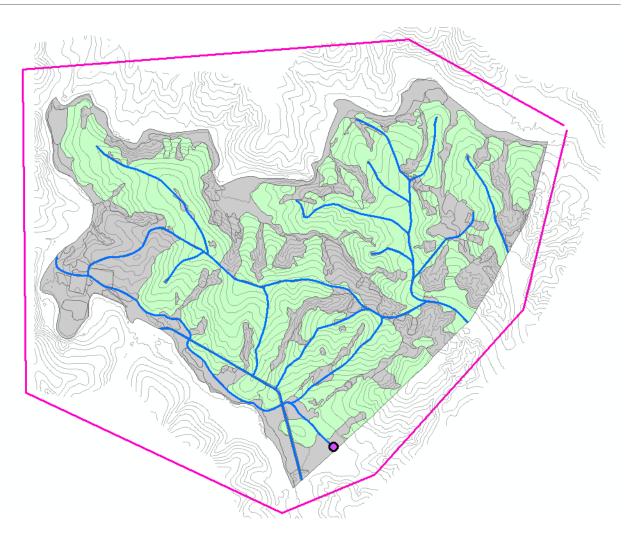


• Feasible Turns:

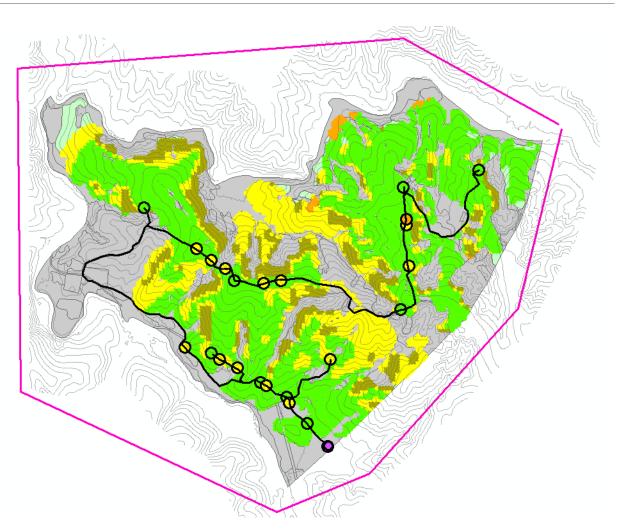




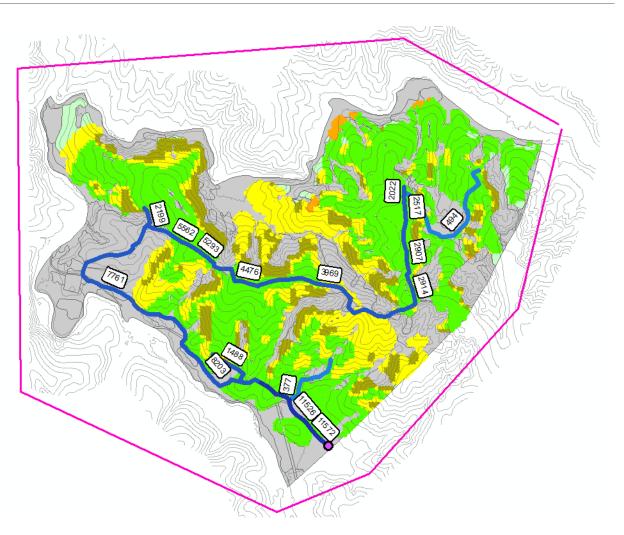
- Example:
- Green areas are plantations.
- The field has existing roads.



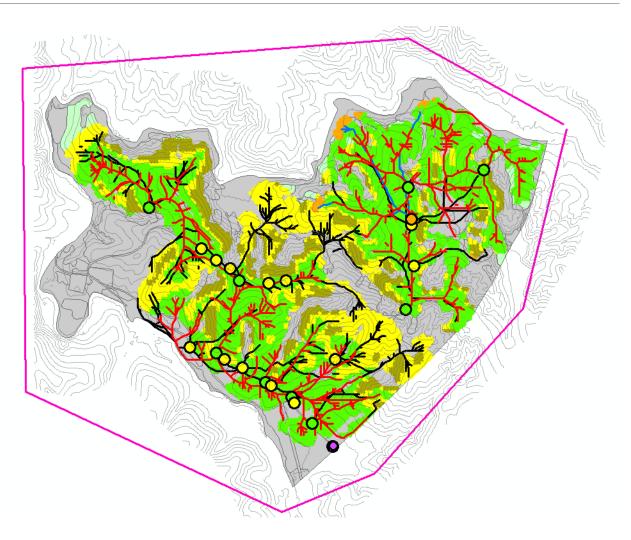
- Solution:
- Different machinery allocation with its corresponding harvesting areas.
- Road network needed for extraction.



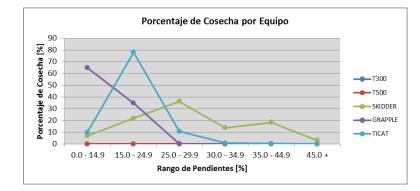
- Solution:
- For each road segment, it shows the timber volume that is transported.
- It is used to determine the amount of gravel needed.

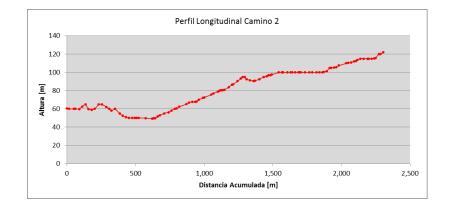


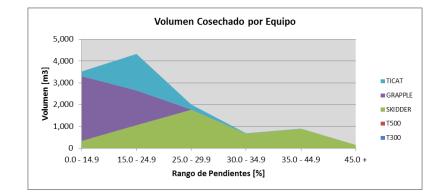
- Solution:
- Skidder harvesting routes.
- It measures the impact of the harvesting in the field.



Tramo	Tipo Camino	Largo	Pendiente	Costo Faja	Costo Ripio	Costo Mant.	Vol Mov Tie	Costo Mov Tie	Flujo Madera	Cto Flu Trans	Cto Per Sue
ID 🔻	*	[Km] 🔻	[%] 👻	[US\$] 🔻	[US\$] 🔻	[US\$] 🔻	[m3] 🔻	[US\$] 🔻	[m3] 🔻	[US\$] 🔻	[US\$] 🔻
1	Nuevo	0.02	0.2	18	0	0	0.43	0	10,659	0	28
2	Viejo Tierra	0.29	6.0	0	0	0	0.00	0	10,659	0	406
3	Nuevo	0.06	2.3	52	0	0	33.69	28	10,289	0	85
4	Viejo Tierra	0.43	5.5	0	0	0	0.00	0	10,289	0	610
5	Nuevo	0.38	4.7	321	0	0	214.43	183	9,777	0	527
6	Viejo Tierra	0.09	5.7	0	0	0	0.00	0	9,543	0	126
7	Nuevo	0.17	5.7	148	0	0	111.36	95	9,543	0	242
8	Viejo Tierra	0.23	8.6	0	0	0	0.00	0	8,090	0	320
9	Nuevo	0.21	4.2	175	0	0	206.82	175	7,328	0	287
10	Viejo Tierra	0.38	1.7	0	0	0	0.00	0	6,303	0	538
11	Nuevo	0.10	4.9	87	0	0	99.88	84	5,425	0	143
12	Viejo Tierra	0.02	4.6	0	0	0	0.00	0	5,425	0	32
13	Nuevo	0.03	4.5	29	0	0	47.88	41	5,425	0	48
14	Viejo Tierra	0.03	9.5	0	0	0	0.00	0	4,908	0	46
15	Nuevo	0.02	0.0	19	0	0	0.00	0	4,908	0	31
16	Viejo Tierra	0.14	4.9	0	0	0	0.00	0	4,908	0	202
17	Nuevo	0.18	5.0	151	0	0	254.09	215	3,705	0	246
18	Viejo Tierra	0.23	3.8	0	0	0	0.00	0	3,705	0	317
19	Nuevo	0.04	6.1	36	0	0	17.77	16	3,112	0	58
20	Viejo Tierra	0.09	3.4	0	0	0	0.00	0	3,112	0	122
21	Nuevo	0.28	4.7	241	0	0	368.71	314	832	0	394
22	Viejo Tierra	0.15	3.6	0	0	0	0.00	0	832	0	208
23	Nuevo	0.02	1.6	21	0	0	13.52	11	832	0	34
	Sub-Total	3.60	4.4	1,298	0	0	1,368.57	1,162	10,659	0	5,050







• Roads:

<ul> <li>Operational:</li> </ul>		
<ul> <li>Existent road:</li> </ul>	2.5	[Km]
<ul> <li>Existent road used:</li> </ul>	1.8	[Km]
<ul> <li>Proposed road used:</li> </ul>	0.2	[Km]
<ul> <li>New road used:</li> </ul>	1.1	[Km]
<ul> <li>Total road used:</li> </ul>	5.6	[Km]
<ul> <li>Earth movement:</li> </ul>	1,320	[m3]

#### • Economical:

<ul> <li>Road maintenance cost:</li> </ul>	900	[US\$]
<ul> <li>Road construction cost:</li> </ul>	1,950	[US\$]
<ul> <li>Gravel cost:</li> </ul>	168,000	[US\$]
<ul> <li>Earth movement cost:</li> </ul>	1,320	[US\$]
<ul> <li>Total road cost:</li> </ul>	172,170	[US\$]

- Harvesting:
  - Operational:

<ul> <li>Total volume:</li> </ul>	30,000	[m3]
<ul> <li>Harvested volume:</li> </ul>	29,000	[m3]
• Total area:	150	[ha]
<ul> <li>Harvested area:</li> </ul>	145	[ha]

#### • Economical:

• Total harvesting cost:

362,000 [US\$]

- KPIs:
  - Roads:
    - Average road cost per road Km [US\$/Km]
    - Average road cost per volume [US\$/m3]
  - Harvesting:
    - Road density [ha/Km]
    - Average harvesting distance [m]
    - Average harvesting slope [%]
  - Total:
    - Average cost [US\$/m3]



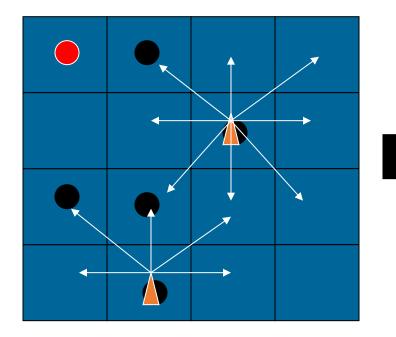
- Benefits:
  - SAVINGS:
    - Fewer roads.
    - Better location of harvesting machinery.
  - ENVIRONMENTAL:
    - Fewer roads.
    - Reduced erosion and water sedimentation.
  - ORGANIZATIONAL:
    - Better analysis quality.
    - Analyst time reduced.
    - It guarantees certifications.



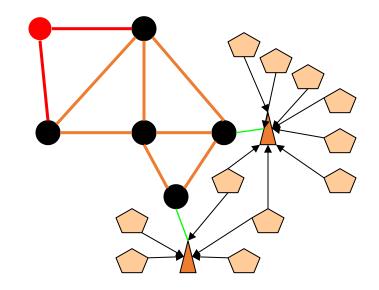


# Optimization Background Theory

#### New Approach: Linear Relaxation



Machines Road Vertices



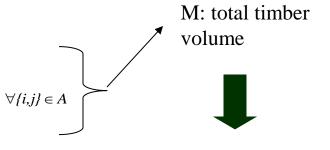
- → Harvesting
- Machinery Installation
- Roads
- 🔶 Timber Nodes

#### New Approach: Linear Relaxation

$$Min \sum_{\{i,j\} \in A} F_{ij} x_{ij} + \sum_{\{i,j\} \in A} c_{ij} f_{ij} + c_{ji} f_{ji}$$

$$f_{ij} \le M \cdot x_{ij}$$
$$f_{ji} \le M \cdot x_{ij}$$

st



$$\sum_{j \in N} f_{ji} - \sum_{j \in N} f_{ij} = \begin{cases} s_i & \text{si } i \in N_T \\ D & \text{si } i = EXIT \\ 0 & \sim & \forall i \in N \end{cases}$$

 $\forall \{i, j\} \in A$ 

Relaxation gives poor results.

 $f_{ij}, f_{ji} \ge 0$ 

 $x_{ii} \in \{0,1\}$ 

 $\forall \{i,j\} \in A$ 

## Multi-commodity Model

- Separate timber from each origin cell into a different commodity.
- Flow in each arc is represented with different variables, one for each commodity:

Fraction of commodity k that

- flows through arc {i,j}.
- Model increases in size.
- Linear relaxation gives good results.

### MC Model (Undirected)

$$Min \sum_{\{i,j\} \in A} F_{ij} x_{ij} + \sum_{k \in K} \sum_{\{i,j\} \in A} (c_{ij}^{k} f_{ij}^{k} + c_{ji}^{k} f_{ji}^{k})$$

Sujeto a

$$f_{ij}^{k} \leq x_{ij}$$
$$f_{ji}^{k} \leq x_{ij}$$

 $\forall \{i, j\} \in A, \forall k \in K$ 

$$\sum_{j \in N} f_{ji}^{k} - \sum_{j \in N} f_{ij}^{k} = \begin{cases} -1 & si \ i = O(k) \\ 1 & si \ i = D(k) \\ 0 & \sim & \\ f_{ij}^{k}, f_{ji}^{k} \ge 0 & \forall (i,j) \in A \end{cases}$$

 $x_{ii} \in \{0,1\}$ 

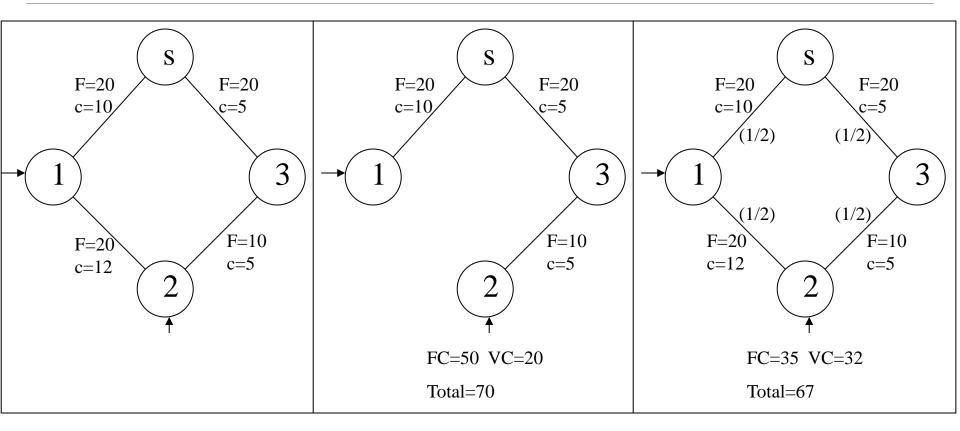
 $\forall \{i,j\} \in A$ 

 $\forall i \in N, \forall k \in K$ 

## Multi-commodity Model

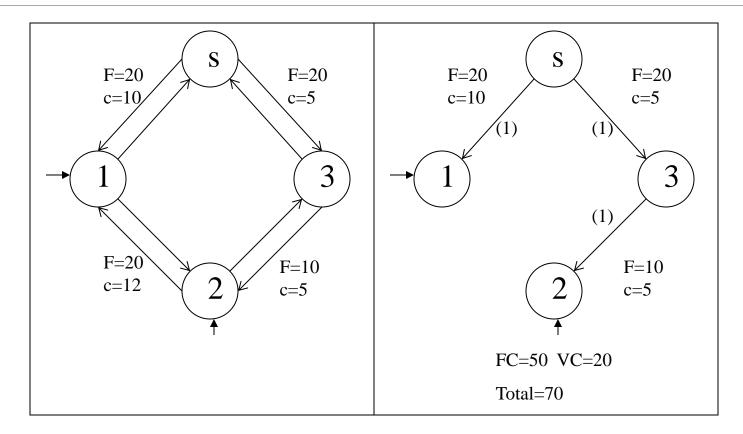
- Model can be improved using special problem features:
  - Uncapacitated network.
  - All commodities share the same destiny.
  - Same transport cost per volume unit.
- Tree-like solutions, one-way flow on each arc.
- **Directed Network** strengthens the model.
- Better solutions obtained.

#### Example



Undirected Network - relaxation produces gap.

## Example



Directed Network - strengthens relaxation

#### **Directed Formulation**

 $Min \sum F_{ij} x_{ij} + \sum \sum c_{ij}^{k} f_{ij}^{k}$  $(i, j) \in A$  $k \in K(i, j) \in A$ 

Sujeto a

 $f_{_{ii}}^{k} \leq x_{ij}$ 

 $\forall (i, j) \in A, \forall k \in K$ 

 $\sum_{j \in N} f_{ji}^{k} - \sum_{j \in N} f_{ij}^{k} = \begin{cases} -1 & \text{si } i = O(k) \\ 1 & \text{si } i = D(k) \\ 0 & \sim \end{cases}$  $\forall i \in N, \forall k \in K$ 

 $f_{ij}^{k} \ge 0 \qquad \forall (i,j) \in A$  $x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A$ 

# **Problem Solving**

#### **Approach: Improve Linear Relaxation**

- $Z_{P} = \text{primal sol.}$  $Z_{E} = \text{opt. integer sol.}$  $Z_{L} = \text{opt. linear sol.}$ = opt. dual sol. $Z_{D} = \text{dual sol.}$

- New Multicommodity formulation improves linear relaxation.
- Model increases rapidly as the problem is scaled, medium and large instances cannot be solved exactly.
- Approximation for the dual problem of the linear relaxation of the model (lower bound).

## **Problem Solving**

- Linear relaxation of the MC model optimally solves smallscale networks, but cannot solve bigger problems.
- **Dual Ascent Procedure**: approximately solves the dual problem of the relaxed MC formulation.
- Medium and Large scale networks can be solved through this approach.

#### Dual Ascent Procedure

- Based on B-M-W (1989).
- Gives as output:
  - Dual Objective Function Value (lower bound).
  - Sub-set of arcs that build a feasible solution of the problem.
- Feasible solution can be improved, obtaining a good solution for the problem (upper bound).
- Quality of the solution can be evaluated using the duality gap.

### Dual Ascent Procedure

 $Max \quad z_D = \sum_{k \in K} v_{D(k)}^k$ 

Sujeto a

$$v_{j}^{k} - v_{i}^{k} \leq c_{ij}^{k} + w_{ij}^{k}$$

$$\sum_{k \in N} w_{ij}^{k} \leq F_{ij}$$

$$\forall (i,j) \in A, \forall k \in K$$

$$w_{ij}^{k} \geq 0$$

$$\forall (i,j) \in A, \forall k \in K$$

Max 
$$v_{D(k)}^k$$

Sujeto a

$$v_{j}^{k} - v_{i}^{k} \leq \hat{c}_{ij}^{k} \qquad \forall (i,j) \in A, \forall k \in K$$

donde

$$\hat{c}_{ij}^{k} = c_{ij}^{k} + w_{ij}^{k}$$

• Fix w-values

- Shortest-path problem, separated by commodity, from node O(k) to D(k).
- Increase the appropriate w-values, in order to increase ZD, the dual OF value.

#### Finding Good Feasible Solutions

- Dual-Ascent: gives a smaller feasible network.
- Two different approaches over this network:
  - Solve the relaxed MC model.
  - Solve NF formulation introducing cuts (row generation).

## Finding Good Feasible Solutions

- Problem can be reduced considerably:
  - Eliminate some "unfeasible" arcs.
  - Eliminate flow balance constraints, as many nodes can't be reached by all commodities.
- Model size reduces considerably, can be solved in medium-large instances.

**P-Dicut** 

$$Min \sum_{\{i,j\} \in A} F_{ij} x_{ij} + \sum_{\{i,j\} \in A} c_{ij} f_{ij} + c_{ji} f_{ji}$$

st

 $\sum_{(i,j)\in\delta+(S)} x_{ij} \geq 1$ 

 $\forall S \subset V, S \cap N_T \neq \phi, EXIT \in (V \setminus S)$ 

 $f_{ij} \leq M_{ij} \cdot x_{ij}$ 

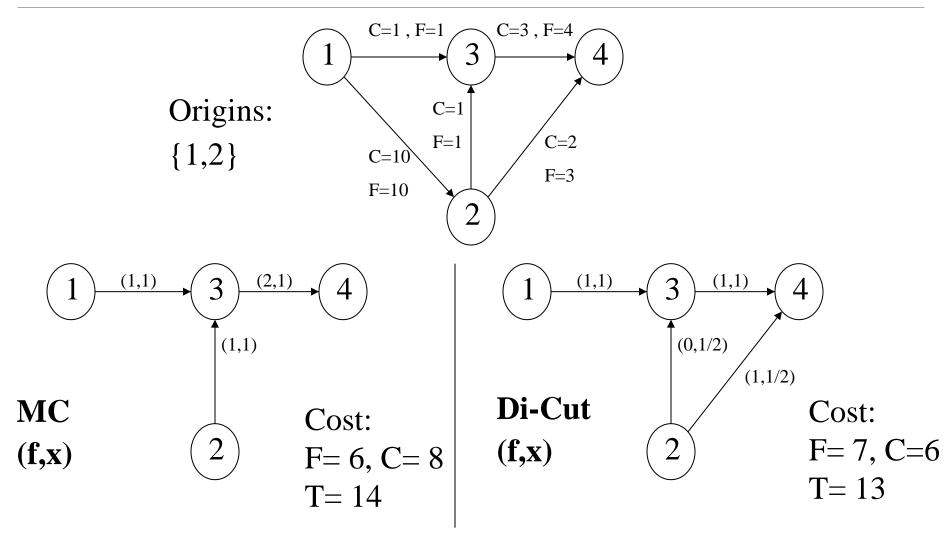
 $\forall (i,j) \in A$ 

$$\sum_{j \in N} f_{ji} - \sum_{j \in N} f_{ij} = \begin{cases} \mathbf{s}_{i} & \text{si } i \in N_{T} \\ \mathbf{D} & \text{si } i = EXIT \\ \mathbf{0} & \sim \\ f_{ij}, f_{ji} \ge 0 & \forall (i,j) \in A \\ x_{ii} \in \{0,1\} & \forall (i,j) \in A \end{cases}$$

## **Di-Cut Formulation**

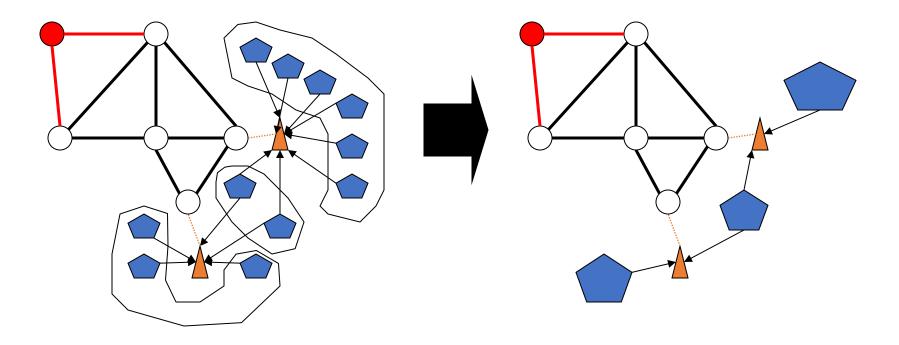
- Model increases exponentially in size.
- Cutting Plane Formulation : detect cut violations with Max-Flow from each origin to EXIT.
- Di-Cut and MC formulation are equivalent if cij=0 (Steiner Tree).
- If cij>0, formulations are not equivalent.
- Heuristic procedure to round up the solution.

#### **Di-Cut Formulation**



#### Network Reductions

#### **Commodity Grouping**



## **Computational Results**

#### **Test Problems**

N٥	Terrain	Timber	Vertex	Possible	Possible	Total Nodes		
	Extension [h]	Nodes	Nodes	Roads	Machines		,	
S	2	25	8	13	9	43		Small
SM	10	1016	14	17	10	1041	ſ	Sman
М	40	3635	106	154	63	3805	}	Medium
L	200	21001	264	364	238	21504	}	Large
Lnr	200	3843	264	364	238	4346	}	Reduced

• Much smaller than real-scale problems (road network is less dense).

## **Computational Results**

N٥	Optimal OF	Dual OF	Dual-Ascent	Feasible (Primal) Solutions		
	Value	Value	Time [secs]	CPLEX	duality gap	iterations
S	4723.50	4723.50	<2	4723.50	0.00%	252
SM	28824.17	28817.42	7	28824.17	0.02%	1321
М	N/A	69935.68	310	70655.93	1.03%	36619
L	N/A	610703.5	6205	624640.81*	2.28%	90308
Lnr	N/A	610802.7	552	624614.6	2.26%	25055

- Gaps below 2.5%.
- CPU time is relatively small.

#### **Computational Results**

#### Results on Row Generation (Dicut Formulation) + Heuristic

Prob	Lower Bound	Dicut	Time (s)	Heuristic	Gap
S	4723.50	4714.22	<1	4938	4.5%
SM	28817.42	28824.2	765	28824.2	0.0%
Μ	69935.68	70656	64525	70656	1.0%
L	610703.5	624590	359046	653007	6.9%

#### Results

- Multicommodity Model:
  - Integer solutions on test problems.
  - Optimally solves small instances.
  - Together with the feasible solution given by the Dual-Ascent solves medium-large scaled problems to 2% of optimality.

### Results

- Dicut Formulation:
  - Can solve larger instances.
  - Requires more CPU time.
- Better than other approaches (Lag. Relax.)
  Lower CPU time, lower gaps, larger scale.

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