INGENIERIA INDUSTRIAL
UNIVERSIDAD DE CHILE

FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS UNIVERSIDAD DE CHILE

Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines

AMSI 2019, Perth

Rafael Epstein
UNIVERSIDAD DE CHILE

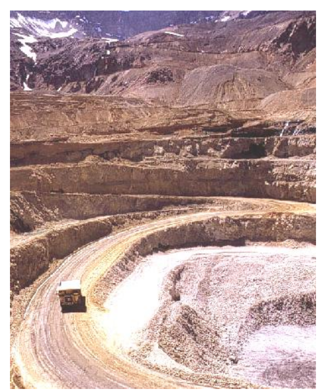
Copper in Chile

- Chile, 5,800 tons, near 40 US$ Billions per year.
- Peru, 2,400 tons, China 1,600 tons, United States 1,200 tons, Congo 1,200 tons, Australia 950 tons.
- CODELCO stated owned company. Largest copper producer in the world.
- El Teniente (under ground) and Chuquicamata (open pit) are two huge, centenary and emblematic copper mines.

Background

- Traditional approach:
  - First planning of mining stage
    (extraction, cut-off grades, tonnage).
  - Then plant stage planning: mills, concentration
    (mill recovery, throughput).
  - Iterations to match supply and demand.
- Challenges:
  - Integration between mining and process stages.
  - Planning multiple mines and plants at the same time.
  - Additional operational constraints.
  - Find the overall optimal operation policy, in the long run (20-50 years).

Problem Description




Overview of the Problem

- **Input:**
  - Mineral resources.
  - Economic & external factors (geo/technological).
  - Regulations/policies.

    (company's objectives, risk constraints).

- **Output:**
  - Extraction plan.
  - Production plan.
  - Investment plan (infrastructure).

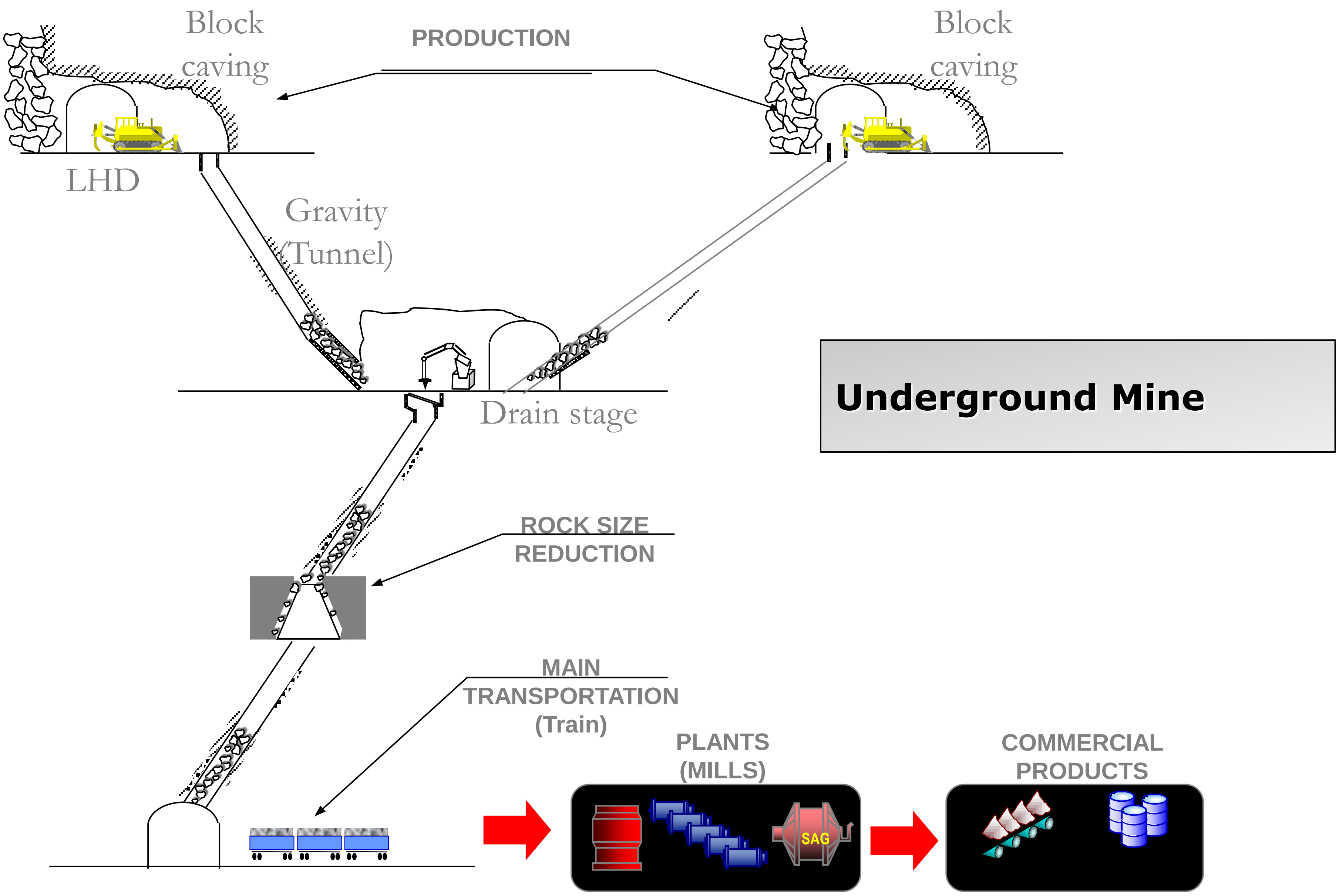
Methodology of Analysis



Optimization Approaches to Long Term Mine Planning

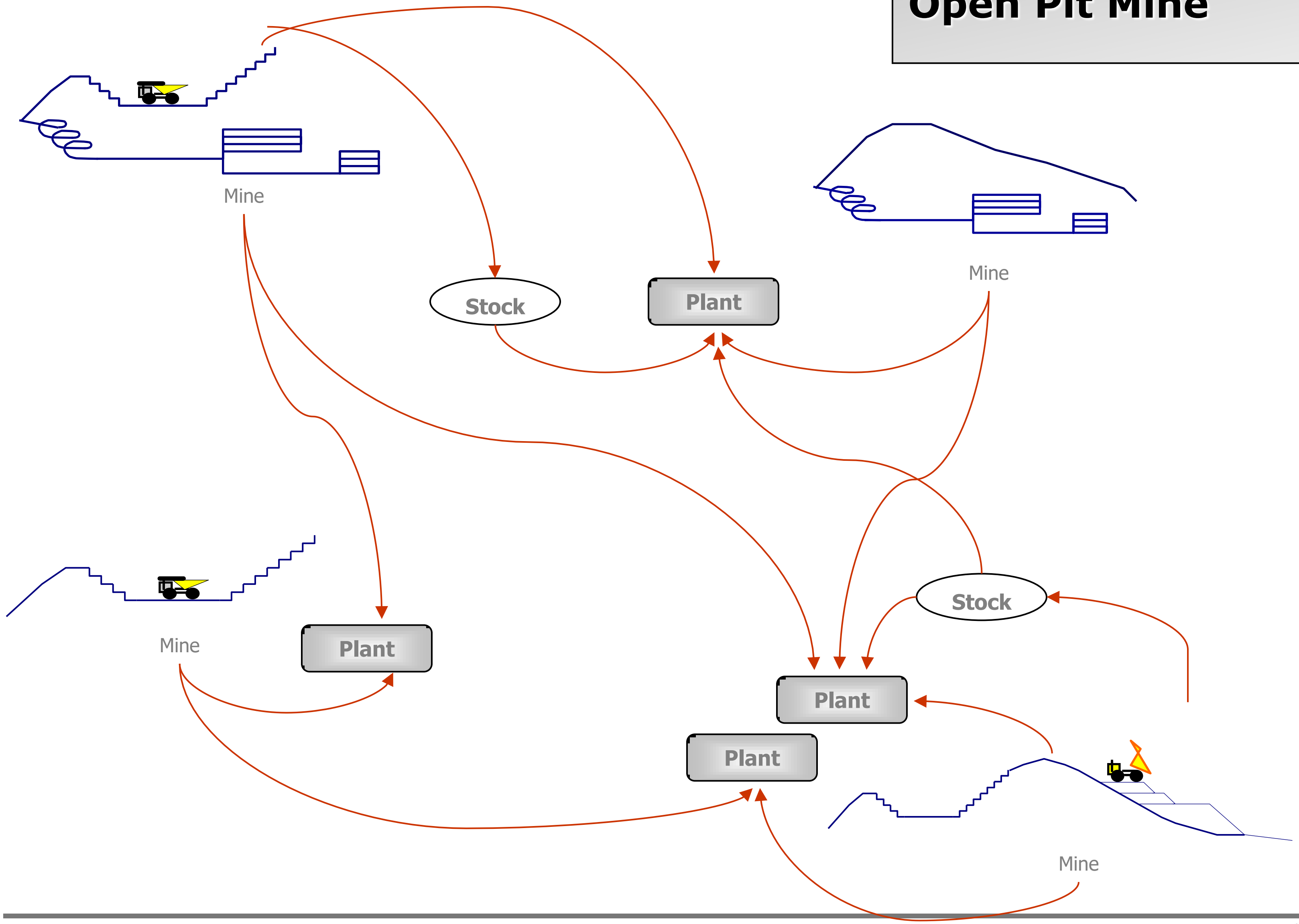
Methodology

- Math programming model (MIP)
  - Optimizes long term plans and large investments in open pit and underground copper mines.
- Model has two main components:
  - Extraction: ore is extracted from the mineral resource.
  - Transportation and processes: ore is processed and refined.
- Multi-commodity capacitated general network flow problem:
  - Capacitated arcs and nodes.
  - Products:
    - Copper
    - Molybdenum
    - Arsenic

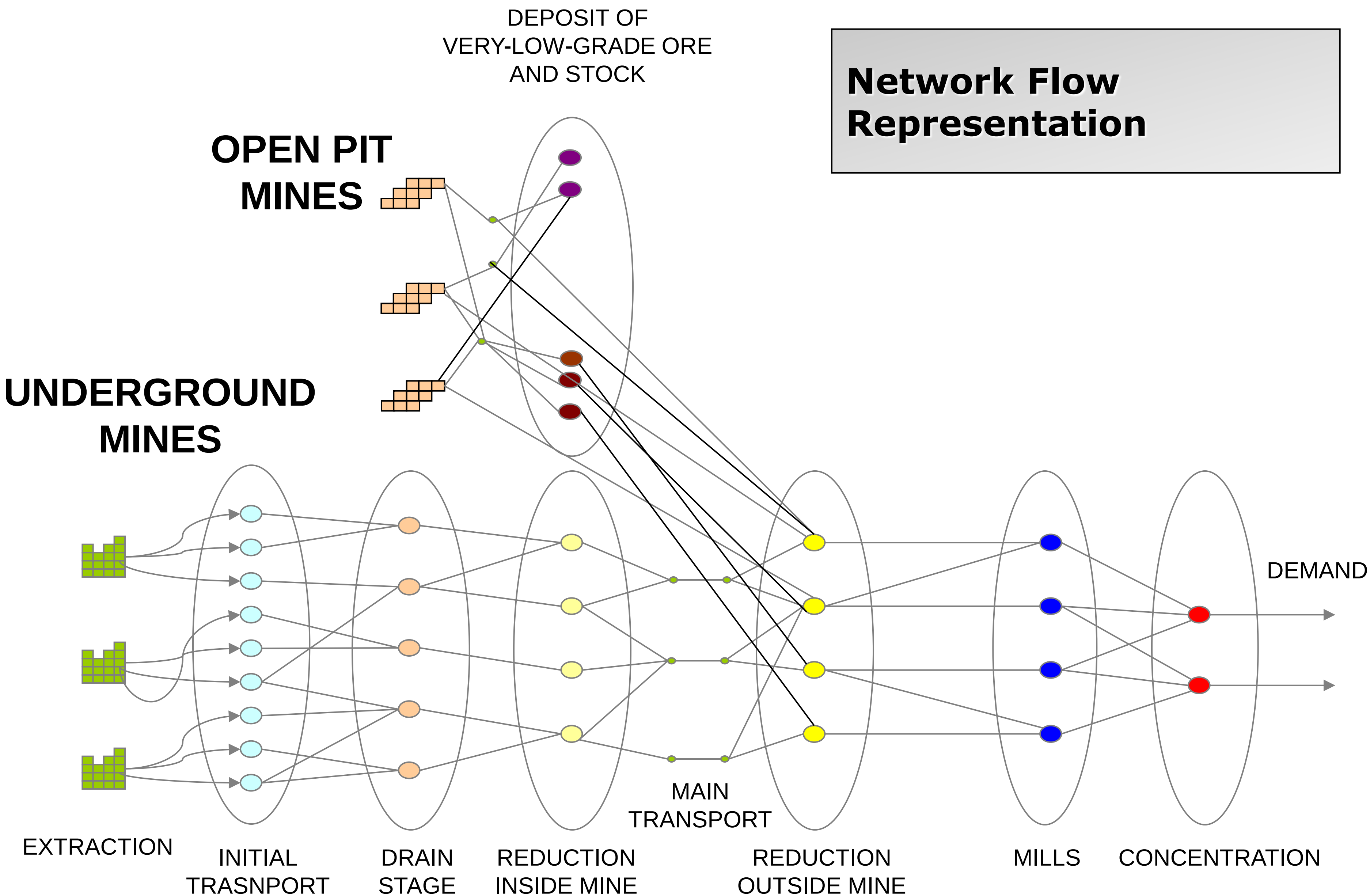


Block caving
PRODUCTION
Block caving
LHD
Gravity (Tunnel)
Drain stage
Underground Mine
ROCK SIZE REDUCTION
MAIN TRANSPORTATION (Train)
PLANTS (MILLS)
SAG
COMMERCIAL PRODUCTS

Open Pit Mine



Network Flow Representation



Underground Mining



Underground mining

Indices (sets)

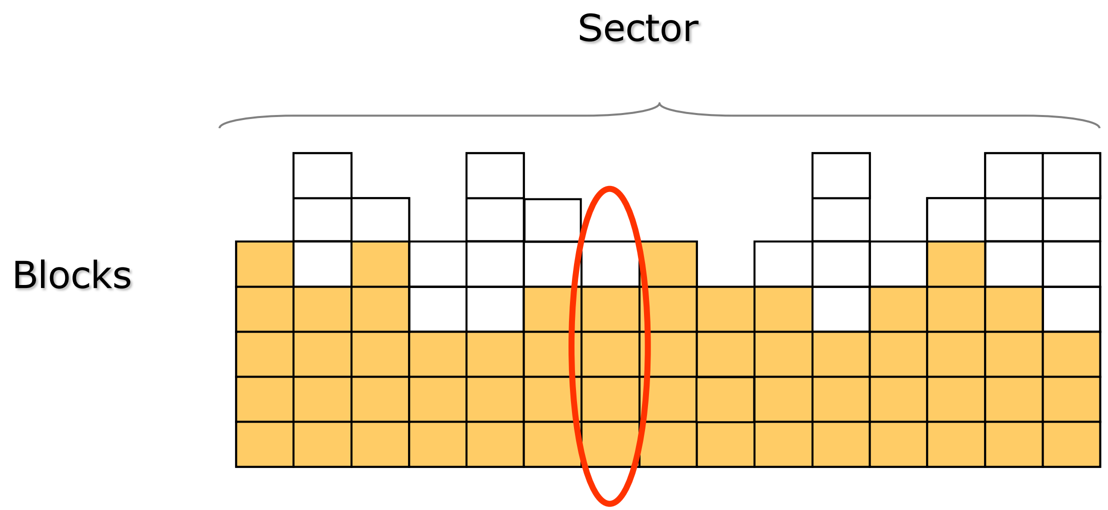
- t : Planning periods
- a : Exploitation sector
- j,i : Mineral Columns
- n : Mineral Block in a column
- v : Process Node

Decision variables

- Extraction at:

  Blocks $\subset$ Columns $\subset$ Sector

- Flow routing



Underground mining

Main Variables

$$Z_{jn}^{t} = \begin{cases} 1 & \text{If block n of the column j is extracted at period t} \\ 0 & \sim \end{cases}$$

$Y_j^t \geq 0$: Height extracted in column j at period *t*

$Ext_a^t \geq 0$: Tonnage extracted (per day) in sector *a* at period *t*

$f_{vpk}^{t} \geq 0$: Flow of product k between node v and node p at period t (generic)

Underground mining

Main Constraints

- Each block can be exploited just once through the horizon.

$$\sum_t Z_{jn}^t \leq 1 \qquad \forall j,n$$

- Blocks must be extracted bottom-up.

$$Z_{j(n-1)}^t \geq Z_{jn}^t \qquad \forall j,n,t$$

- Sequence in which columns can be extracted.

$$Z_{j0}^t \geq Z_{i0}^t \qquad \forall (j,i) \in SEC(j,i)$$

Underground mining

Main Constraints

- Relation between blocks, columns and sectors.

$$Y_j^t = Y_j^{t-1} + \sum_{n \in N(j)} Z_{jn}^t \cdot h_{jn} \qquad Ext_a^t = \frac{\sum_k \sum_{j \in J(a)} \sum_{n \in N(j)} Z_{jn}^t \cdot A_{kjn}}{DP_a^t}$$

- Sector extraction capacity.

$$MIN\_EXT_a^t \leq Ext_a^t \leq MAX\_EXT_a^t \qquad \forall a, t$$

- Time availability constraint (at max extraction rate).

$$\sum_{n \in N(j)} \left(Z_{jn}^t \cdot \beta_{jn} \right) \leq DP_a^t \qquad \forall\, t, j \in J(a)$$

Underground mining

Main Constraints

- Maximum allowed horizontal extraction per sector (geotechnical constraint).

$$MIN\_AREA_a^t \leq \sum_{j \in J(a)} Z_{j0}^t \cdot area_j \leq MAX\_AREA_a^t \qquad \forall a,t$$

- Flow conservation and node/arc capacities.

$$f_{vk}^t = \sum_{p|(v,p) \in VP} f_{vpk}^t$$

$$\sum_{k \in K_V} f_{vpk}^t \leq CAP_{vp}^t \qquad K_V \text{ : products at node } v$$

- Maximum level of allowed contaminants.

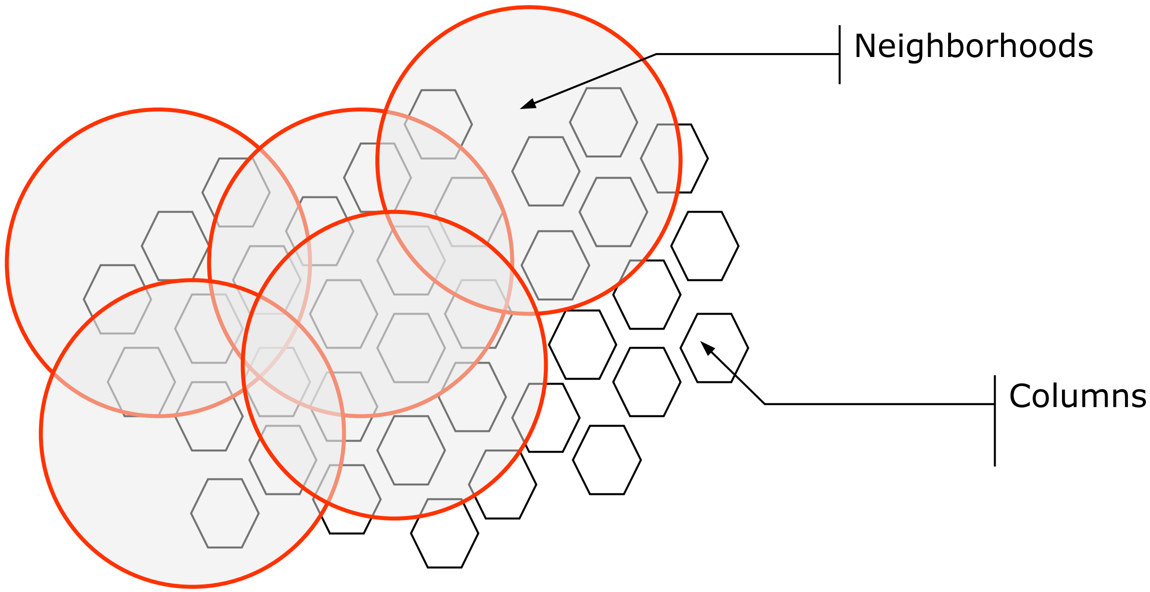
$$\sum_o \sum_k CT_{okk'} \cdot f_{omkt} \leq MaxC_{k'} \quad \forall k' \in CONT, \forall t, \forall m$$

Underground mining

- Regularity in heights

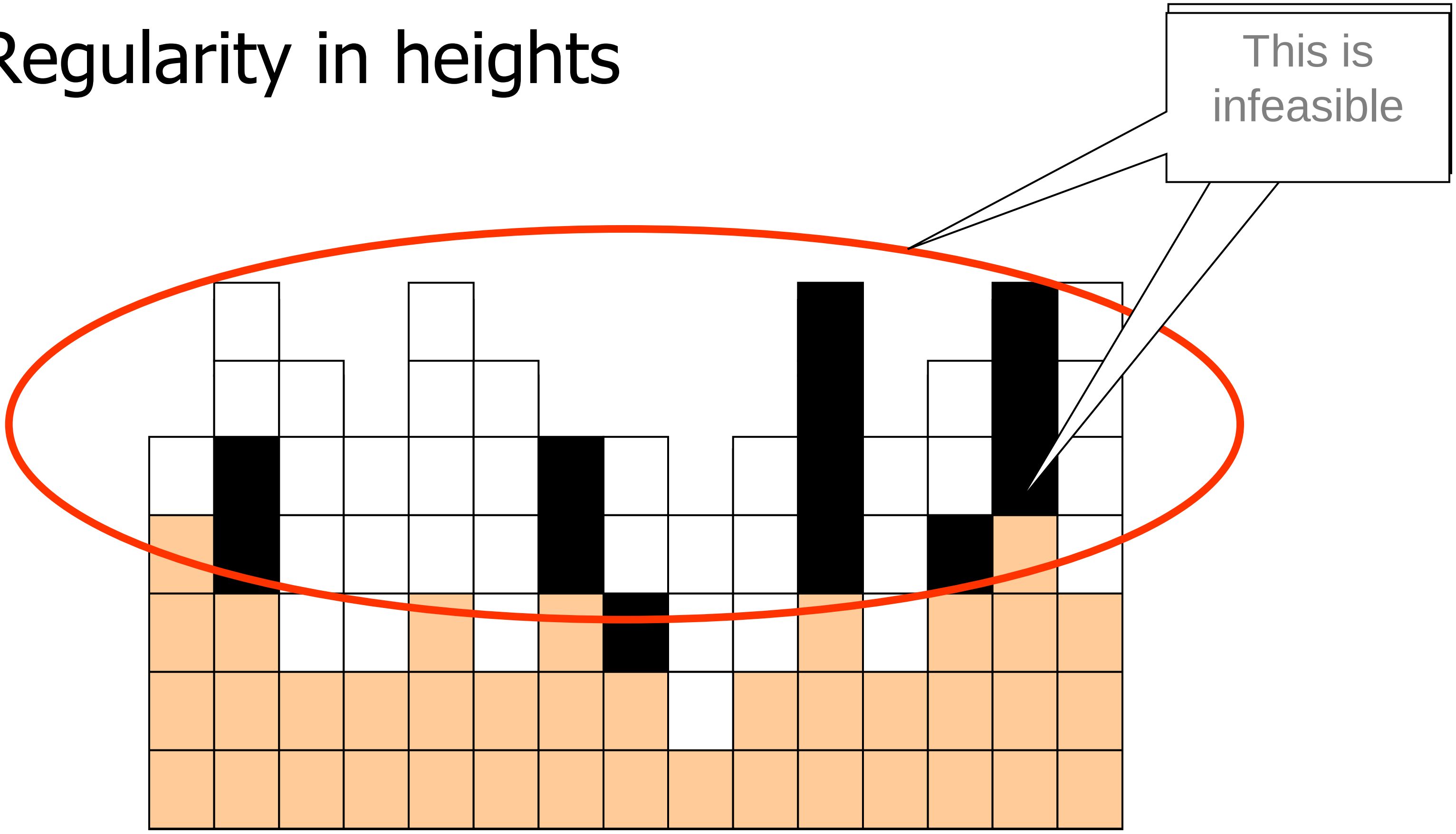
$$Y_j^t - Y_i^t \le \delta_{ji}^a \qquad \forall (j,i) \in JI \qquad \forall t$$

- Interaction with neighborhoods

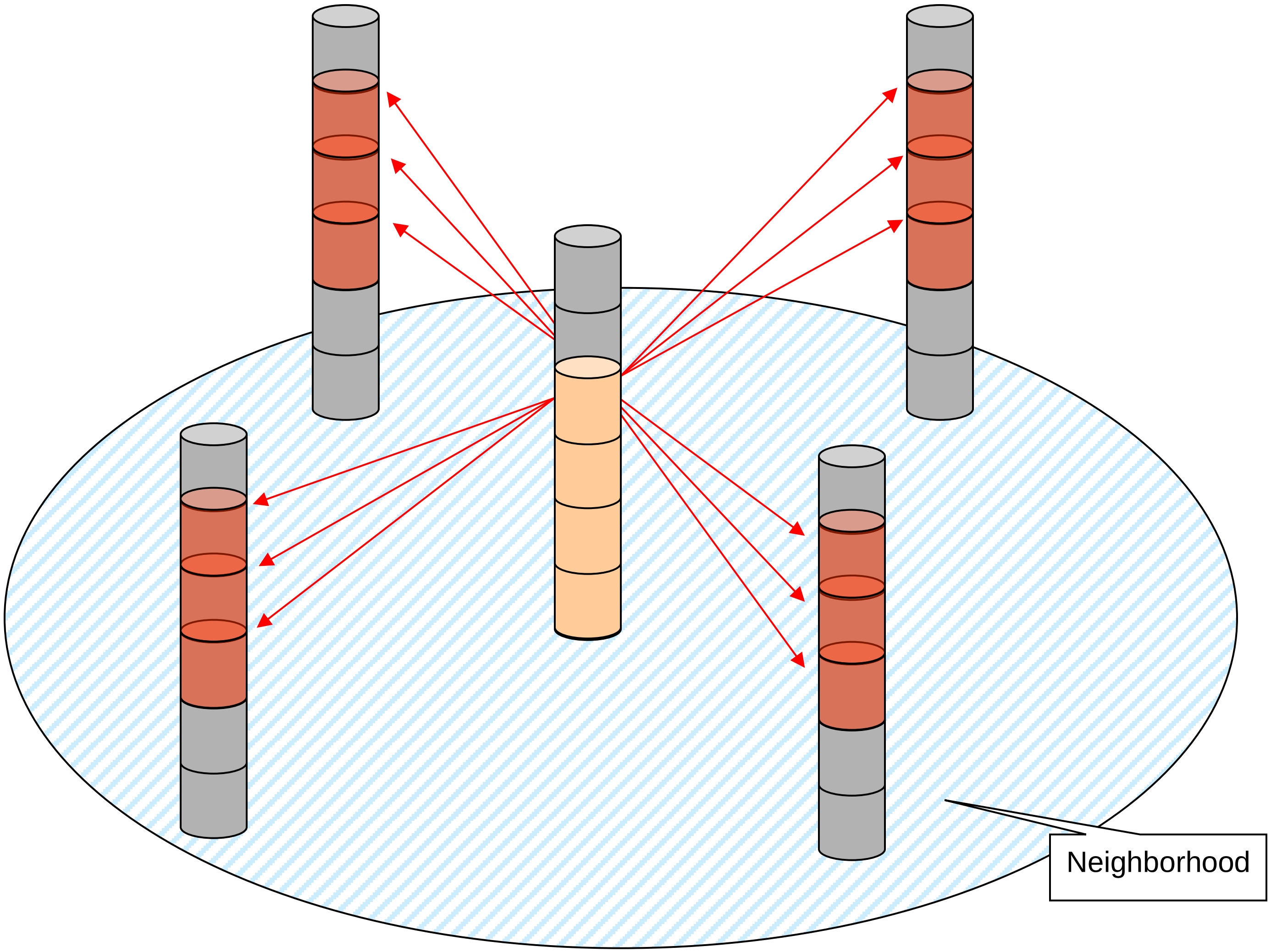


Underground mining

- Regularity in heights



Underground mining



Objective Function

- Goal: To Maximize Net Present Value
  - Income: sale of products and sub-products.
  - Cost: Extraction, transportation and process at plants.
  - Investment: New projects .

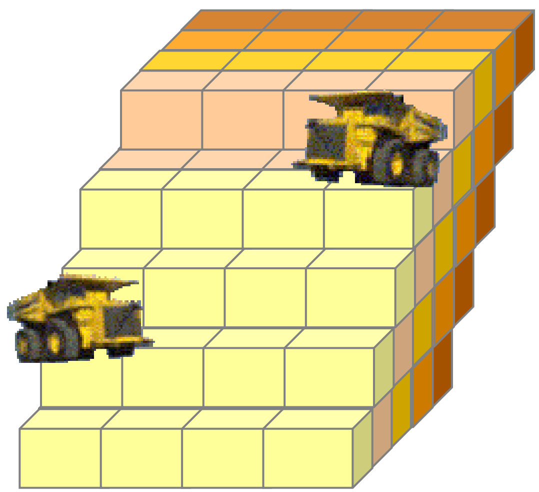
$$\max \; O.F. = I - FC - VC$$

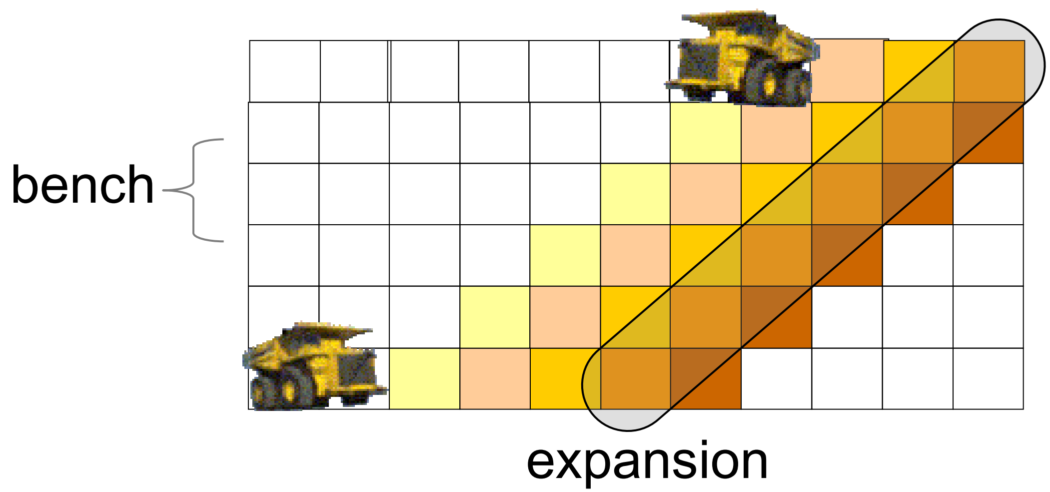
Open Pit Mining


Optimization Approaches to Long Term Mine Planning

Open Pit Mining

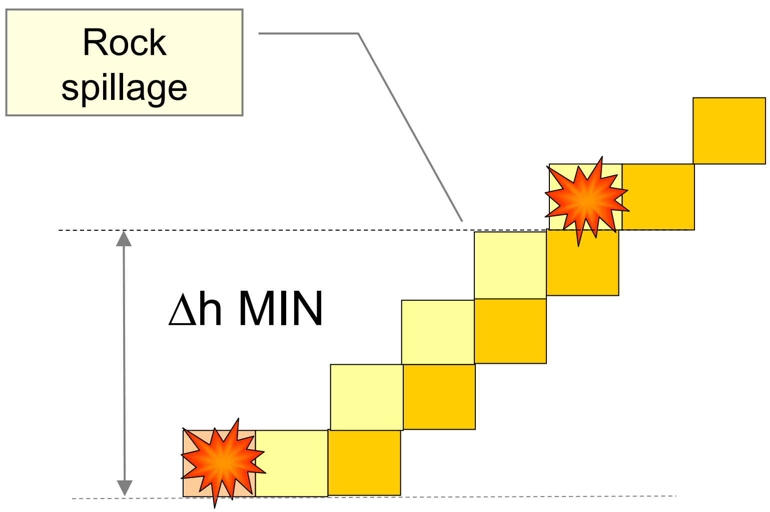
- Mining resource:
  - Expansion: exploitable "slice" of the pit.
  - Bench: transversal cut, characterized by elevation and height.

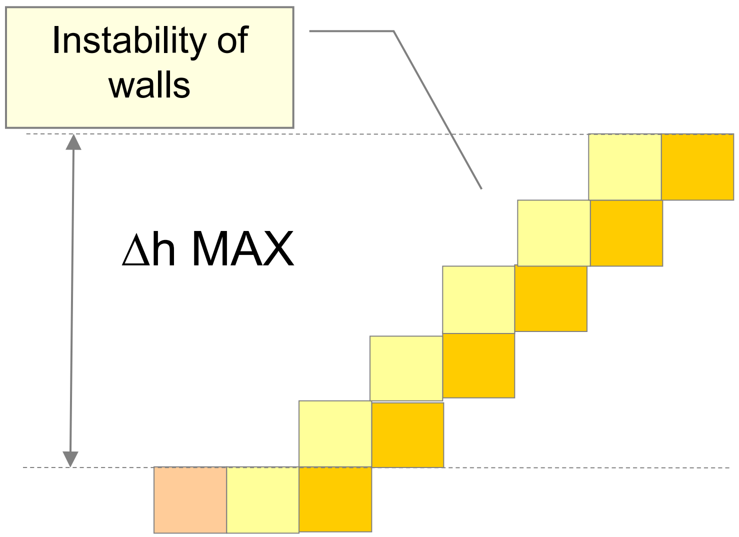




Open Pit Mining

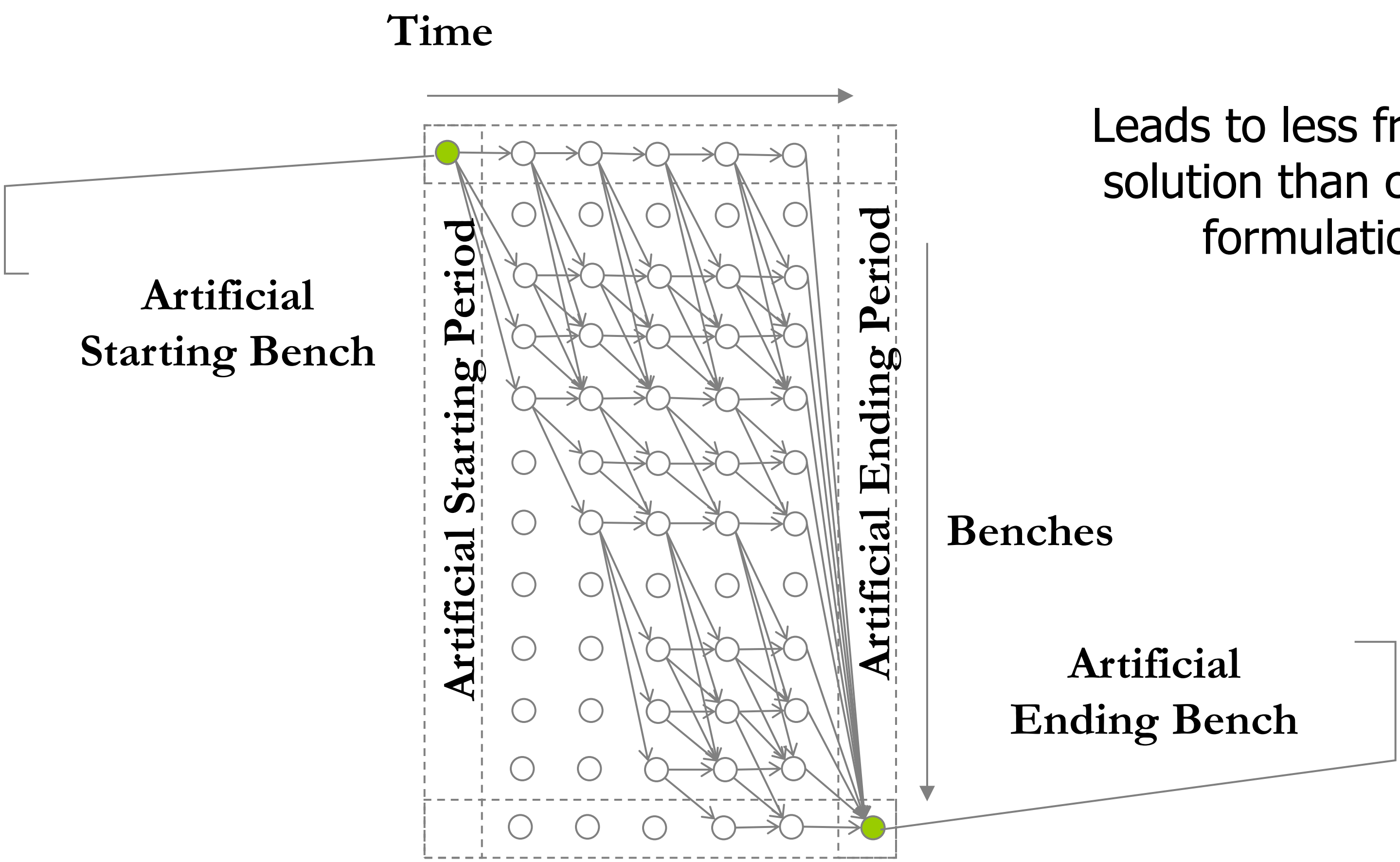
- Operational constraints $\Rightarrow$ Min/Max extraction rates
- Only resource on the surface can be exploited.
- Safety constraints.





Network Flow Formulation

Network flow formulation to extract a sequence of benches (includes safety considerations).



Leads to less fractional solution than classical formulation.

Network Flow Formulation

- **Main decision variables**

  w_{ijt} = 1 if benches from (i+1) to j are extracted on period t

  $z_{it} \in \{0,1\}$ equals 1 if bench i is extracted in period t

- **Sets**

  Or(a): artificial starting bench.

  De(a): artificial ending bench.

  Pini: artificial starting time period.

  Pfin: artificial ending time period.

  SU(i,t): successors of node (i,t).

  AN(i,t): predecessors of node (i,t).

(1) Flow conservation:

$$\sum_{h \in AN(i,t)} w_{hit} = \sum_{j \in SU(i,t)} w_{ij(t+1)} \quad \forall i, \forall t$$

(2) Each expansion can start only one extraction sequence.

$$\sum_{j \in SU(Or(a),0)} w_{Or(a)j1} = 1 \quad \forall a$$

(3) Each expansion can end only one extraction sequence.

$$\sum_{j \in a \cup \{Or(a)\}} w_{jDe(a)Pfin} = 1 \quad \forall a$$

(4) Relation between variables.

$$z_{it} = \sum_{j \in SU(i,t)} \sum_{h<i} w_{hjt} \quad \forall i, \forall t$$

Open Pit Mining

- Downstream processes:
  - Same as underground but with several alternatives routes.
  - Different types of plants: flotation, leaching, bioleaching, low-grade sulfides.
  - Stocking areas: large amount of material is stored for future use.
  - Waste dumps: material without economic value.

Solving the Model

- Real instances lead to hard problems (old instances).

Model	Nº Constraints	Nº Variables	0-1 Variables
Underground mine	446,521	535,639	196,386
Open pit mine	245,391	898,742	160,386

- Given the large amount of binary variables, especially those representing the extraction stage, solving the model using a mixed integer programming routine is nonviable.

Rounding Heuristic

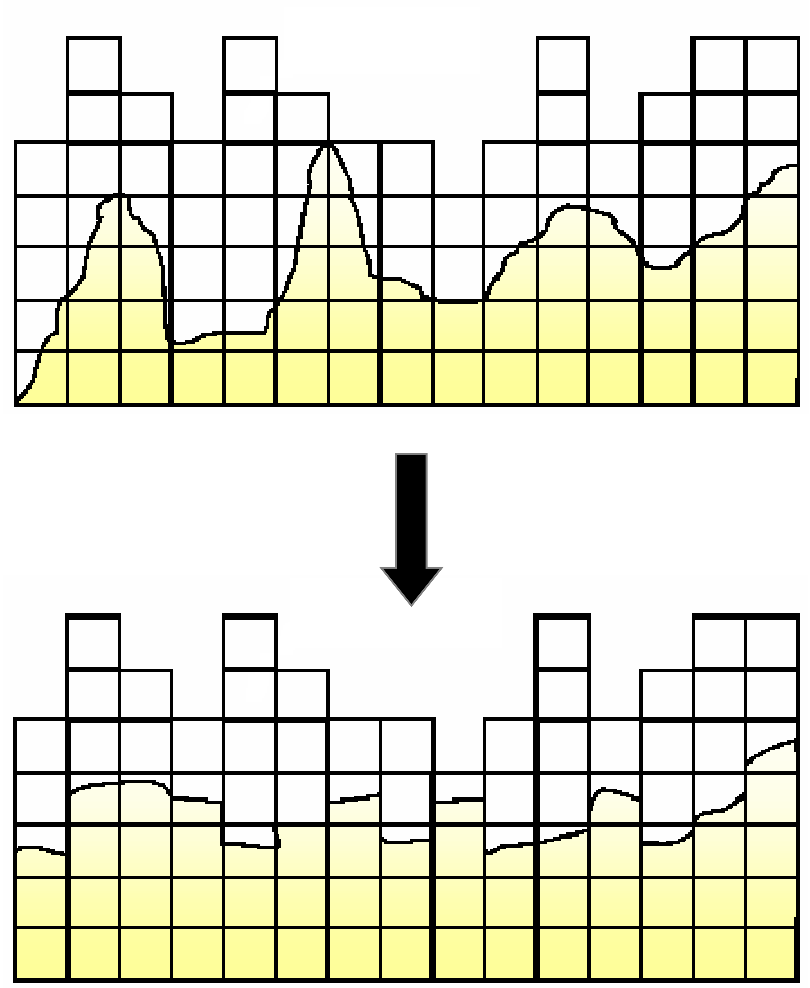
- The integrality condition is relaxed and a continuous version is solved:

$$Z_{jn}^{t} \in \{0,1\} \quad \longrightarrow \quad 0 \leq Z_{jn}^{t} \leq 1$$

- Heuristic consists of fixing binary variables based on the solution of continuous version.

Rounding Heuristic

- Logic based on:
  - Tonnage per column to remain constant.
  - Fix variables in logical order.
- This process is iterative and stops when the solution is integer.
- Since copper grade is better for lower altitude blocks, LP is reasonably good.
- A similar heuristic is used for open pit mine problems.



Applications in CODELCO

Applications in Codelco

- The model was implemented using GAMS and was solved using CPLEX.

- System implementation at CODELCO’s Divisions:
  - El Teniente.
  - North Division.
  - El Salvador.
  - Andina Division.

- When using our methodology, the NPV of projects have increased up to 8%.

An Example in El Teniente Mine

	Base Plan (Reference) [MUS$]	Optimization Model [MUS$]	Difference [MUS$]	
Income	6479.7	6637.6	157.9	2.4%
Costs	3279.9	3320.0	40.1	1.2%
Investments	870.9	870.9	0.0	0.0%
NPV	**2328.9**	**2446.7**	**117.8**	**5.1%**

- Optimization model utilizing same investment plan as reference case
- Model with fixed sequencing of columns
- When relaxing sequencing of columns improve NPV to 6.1%

Model Evaluation – An Example in North Division

	Model Integrated [%]	Legacy Independet [%]	Model Independent [%]	Legacy Integrated [%]
Income	1.4	0.0	1.7	1.1
Costs	-1.6	0.0	0.8	0.0
Investments	-27.3	0.0	-31.8	0.0
NPV	**8.2%**	**0.0%**	**4.7%**	**3.2%**

See: “Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines”, Operations Research, Volume 60, Number 1, 2012.

INGENIERIA INDUSTRIAL
UNIVERSIDAD DE CHILE

FACULTAD DE CIENCIAS
FÍSICAS Y MATEMÁTICAS
UNIVERSIDAD DE CHILE

Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines