A bilevel programming model for market regulation: Application to the Mexican petrochemical industry

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Bilevel Optimization Problem

$$
\begin{array}{ll}
\max\limits_{x,y} & f(x,y) \\
\text{s.t.} & (x,y) \in X \\
 & y \in S(x) \\
\text{where} & S(x) = \operatorname*{argmax}\limits_{y} g(x,y) \\
 & \text{s.t.} (x,y) \in Y
\end{array}
$$

Adequate framework for Stackelberg game

- Leader: 1st level,
- Follower: 2nd level,
- Leader takes follower's optimal reaction into account.

Heinrich von Stackelberg
(1905 - 1946)

First paper on bilevel optimization

Bracken & McGill (OR,1973): First bilevel model, structural properties, military application.

Mathematical Programs with Optimization Problems in the Constraints

Jerome Bracken and James T. McGill

Institute for Defense Analyses, Arlington, Virginia

(Received October 5, 1971)

This paper considers a class of optimization problems characterized by constraints that themselves contain optimization problems. The problems in the constraints can be linear programs, nonlinear programs, or two-sided optimization problems, including certain types of games. The paper presents theory dealing primarily with properties of the relevant functions that result in convex programming problems, and discusses interpretations of this theory. It gives an application with linear programs in the constraints, and discusses computational methods for solving the problems.

Applications

- Economic game theory
- $\boxed{\text{Production planning}}$
- Revenue management
- Security
- …

Historical background

1958 - 2014: the Mexican legal framework divided the petrochemical industry into two branches.

- Basic petrochemical. Economic activities that transform the natural gas and oil in raw materials (for example, methane, ethane or naphtha); monopole of state-owned firms: Petroleos Mexicanos (PEMEX) and its subsidiaries or associated firm.
- Secondary petrochemical. Used these supplies to make oil derivatives (methanol, ethylene, ammonia) for other industries;
  private and government firms compete.
-

Problem description

A more general situation with two vertically integrated industries:

- First industry: state-owned firm, monopole for the production of raw materials (supplies) for the second industry.
- Second industry: private firms + state-owned firm compete to produce commodities.

- All firms have a limited production capacity.
- The state-owned firm must achieve a minimum income.

Related literature

Merrill and Schneider (1966):

- idea of regulating market by using state-owned firms.

Harris and Wiens (1980), Sertel (1988), Cremer, Marchand and Thisse (1989), De Fraja and Delbono (1989), (1990), Nett (1993):

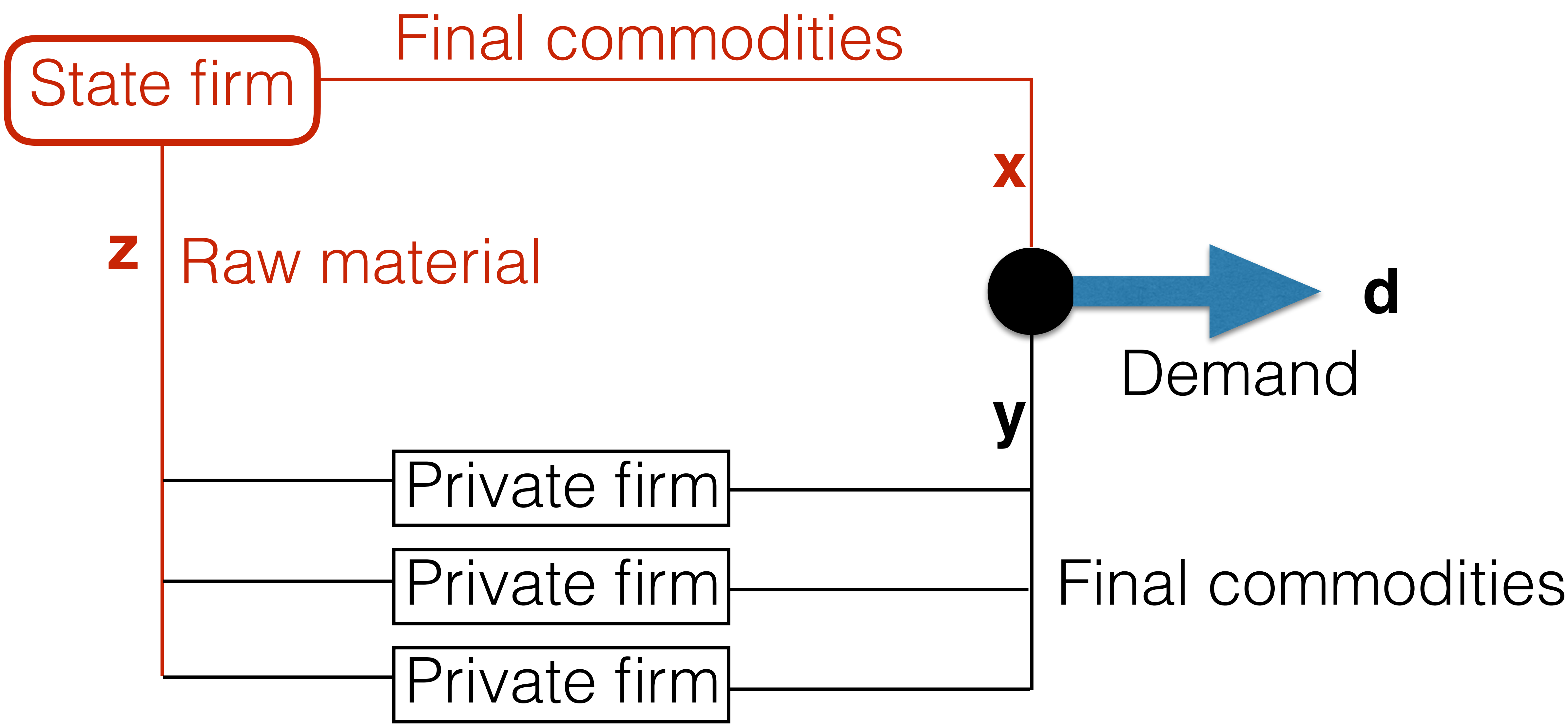
- developed the idea of regulation by participation
- basis of the framework for mixed oligopolies.

Problem description

- The leader is the government. Its goal is to regulate the market: balance supply and demand for commodities.

- The follower is a regulatory organization of the private firms. Its goal is to allocate the supplies to maximize the total profit of private firms.

Problem description



Assumptions

a. Static analysis

b. Complete information

c. Only one important supplier for each commodity

d. Linear production and cost

e. Demand and prices are given

f. Optimistic approach

Bilevel formulation

$$
\begin{array}{rrcll}
\multicolumn{5}{l}{\displaystyle\min_{x,z,r,s,y} \sum_{i\in I}(r_i + s_i)} \\
\text{subject to: } \dfrac{\sum_{j\in J} y_{ij} + x_i}{d_i} + r_i - s_i & & = & 1 & \forall i \\
 & t & \leq & \displaystyle\sum_{i\in I}(p_i - c_i^G)x_i & \\
0 \leq & x_i & \leq & q_i^A & \forall i \\
0 \leq & z_i & \leq & q_i^B & \forall i \\
 & r_i & \geq & 0 & \forall i \\
 & s_i & \geq & 0 & \forall i \\
 & y & \in & \displaystyle\operatorname{argmax} \sum_{j\in J}\sum_{i\in I}(p_i - c_{ij}^E)\hat{y}_{ij} & \\
 & & & \text{subject to: } \displaystyle\sum_{j\in J} a_{ij}\hat{y}_{ij} \leq z_i \quad \forall i & \\
 & & & \qquad\qquad\qquad \displaystyle\sum_{i\in I} b_{ij}\hat{y}_{ij} \leq m_j \quad \forall j & \\
 & & & \qquad\qquad\qquad \hat{y}_{ij} \geq 0 \quad \forall i, \forall j &
\end{array}
$$

Useful observations

- The second level problem is linear
- Its dual feasible region does not depend on the first level decisions

Solving the problem exactly

- Reformulate problem as single level

  ✳ Leader's objective
  ✳first and second level constraints
  ✳ constraints of second level dual

  linear

  ✳complementarity constraints of second level: **non linear**

➡ Introduce binary variables

➡ MILP

Heuristic approach

In an optimal solution, the primal and dual solutions of the second level are vertices with same value

Gauss-Seidel approach:

$$
\begin{array}{c}
\boxed{\text{Restricted Master (SOME vertices of second level dual)}} \\
\mathbf{z} \downarrow \qquad \uparrow \boldsymbol{\alpha}, \boldsymbol{\beta} \\
\boxed{\text{Second level dual}}
\end{array}
$$

Computational experiments

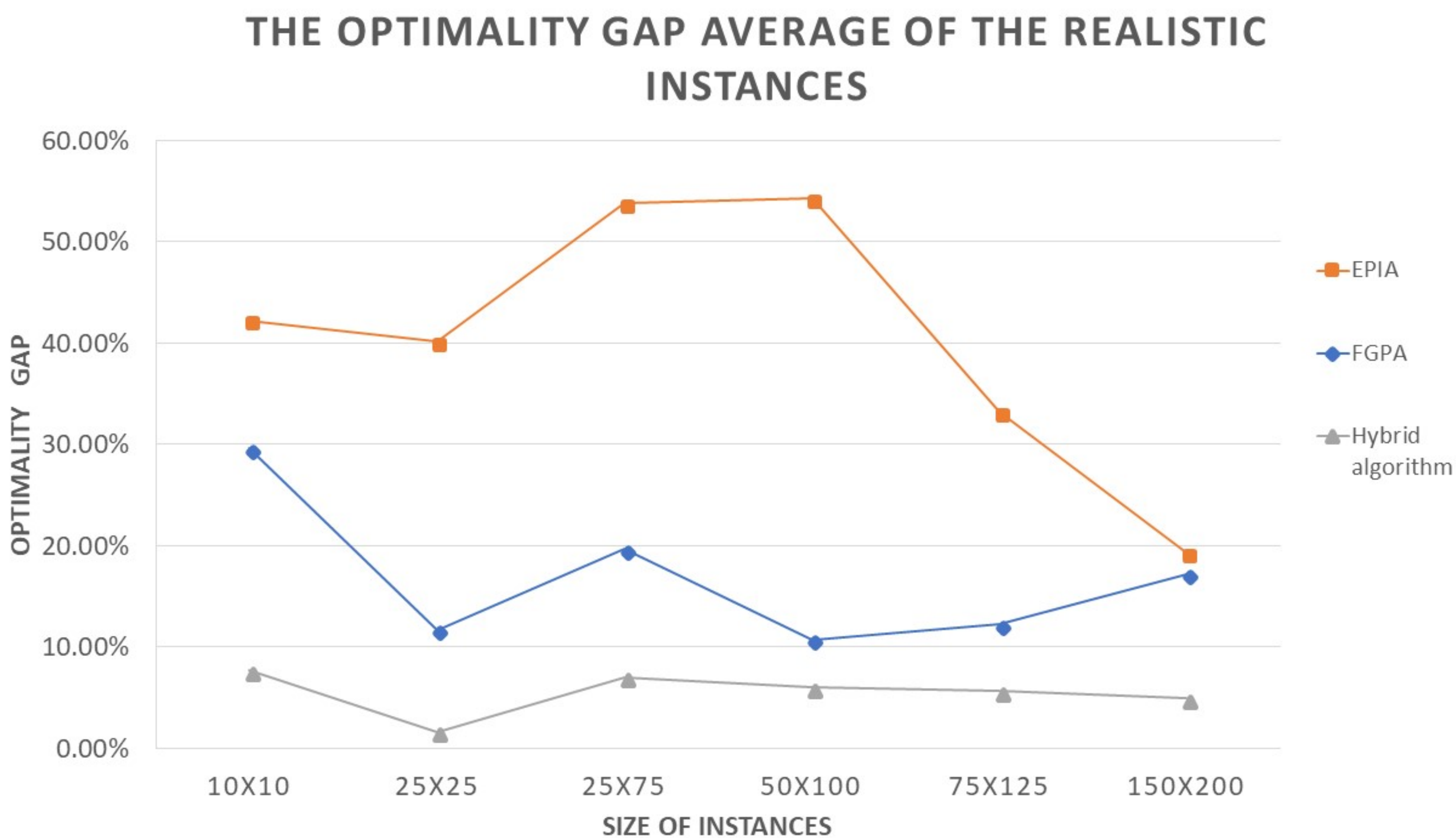
- Reformulation (MILP) and the heuristics were implemented in C++ and solved with CPLEX.
- Realistic instances use the values of the Secretary of Economy, the INEGI, the PEMEX and the information of the BVM of six Mexican petrochemical firms.
- 30 instances for each size.

Computational experiments

Table: Average of the objective value and solution time for the realistic instances

Size	Ref. (MILP)		EPIA		FGPA		Hybrid Algorithm	
	O.V.	CPU(s.)	O.V.	CPU(s.)	O.V.	CPU(s.)	O.V.	CPU(s.)
$\|I\| = 10, \|J\| = 10$	0.506	0.18	0.774	0.23	0.825	0.16	0.524	0.41
$\|I\| = 25, \|J\| = 25$	1.000	1.86	1.570	0.30	1.171	0.35	1.020	0.71
$\|I\| = 25, \|J\| = 75$	0.527	4.36	1.025	0.48	0.565	0.39	0.552	1.02
$\|I\| = 50, \|J\| = 100$	1.051	53.15	2.041	2.39	1.132	1.14	1.091	5.18
$\|I\| = 75, \|J\| = 125$	1.546	268.89	2.339	8.89	1.811	3.68	1.621	20.69
$\|I\| = 150, \|J\| = 200$	3.279	2754.68	4.129	61.82	5.037	12.06	3.443	112.25

Computational experiments



Computational experiments

Table: Relative reduction of time

Size	EPIA	FGPA	Hybrid Algorithm
10x10	12%	15%	-48%
25x25	746%	534%	190%
25x75	1156%	1101%	445%
50x100	3418%	5418%	1176%
75x125	5269%	10696%	1499%
150x200	9210%	30206%	3324%

Conclusion

- Bilevel optimisation: rich and adequate framework for market regulation
- Computationally challenging
- Exploit problem structure
- Research avenues: Nash equilibrium in second level


Thank You
Arigato
Merci
Grazie
Spasiba
Cheers
Efharisto
Danke
Gracias
Hvala
Toda