Simultaneous Stochastic Optimization of Mining Complexes / Mineral Value Chains
A review of applications, solution methods and key findings

Amina Lamghari
Roussos Dimitrakopoulos


Introduction & Basics
Conventional long-term planning
Orebody models
Major limitations
Stochastic workflow
Quick Introduction to Mining

• From small holes to big pits: **drill** for a new deposit.

Quick Introduction to Mining

• From small holes to big pits: **assay** the drill cores ("samples") for metal content ("grade", %, gpt, ppm)

- 2.0% (High)
- 0.7% (Medium)
- 0.1% (Low)
- 0.0% (Barren)
• From small holes to big pits: define the mineralized volume (**orebody**).

Quick Introduction to Mining

• From small holes to big pits: **discretize** the model into 3D volumes (often, but not necessarily, **blocks**).

Quick Introduction to Mining
Quick Introduction to Mining

• From small holes to big pits: create models for the metal content (grades) for each of the volumes (blocks) in the orebody ("geostatistics").

2.0% (High)  
0.7% (Medium)  
0.1% (Low)  
0.0% (Barren)

Conventional / Deterministic Workflow

- Orebody Modelling
- Mine Design & Production Scheduling
- Financial & Production Forecasts

Estimated Orebody Model  
Deterministic Design  
Production Forecasts

Can a single estimated model represent a mineral deposit?  
(Grade variability, uncertainty)  
Is this design the optimal / best?  
Are we able to meet expected forecasts?
Deterministic Workflow

~80% of Failures Due to Geological Risk

Australasian Examples – Technical Risk

% Deviation

Mining Decisions

Attributes of Interest

Au grade
CO$_3$ grade
SS grade
SS/CO$_3$ ratio
Rec(Autoclave)
Rec(Leach)
Rec(Mill)
Tonnage

Oxide Mill
Oxide Leach
Autoclave
Stockpiles
Waste Dump

2016
2017
2018
2019
2020

Traditional production scheduling methodologies neglect uncertainty and variability!

Source: M. Godoy, Newmont Gold, SME 2016
Traditional Orebody Models: Some Limitations and *Shortcomings*

Conventional models **DO NOT account for uncertainty**….

Estimation methods try to approximate some average grade value ... not the actual one.

**Grade legend**
- >10.0
- >5.00
- >1.50
- >0.60
- >0.35
- >0.01

**Estimation vs Simulation**

**Quantifying Uncertainty**

- **Simulated Orebody Models.** This is a Monte Carlo type simulation …

*Estimated Orebody Model*

A mature, well drilled and understood gold deposit

3 simulated scenarios of the same section (SMU grade)
Traditional Orebody Models - Limitations & Shortcomings

Simulated grades

*Estimated ( - - - , - - - ) vs simulated models ( – – – ) as inputs to …*

The representation of a mineral deposit and related pertinent attributes – estimated vs simulated - MATTERS …

Traditional Orebody Models - Limitations & Shortcomings

Bench in a gold deposit being mined

Black indicates DDH grade above 1.3 g/t and grey between 0.7 and 1.3 g/t

Real blast hole data

Real mineral deposits are highly variable, not smooth

10x10x5m blocks
Traditional Orebody Models - Limitations & Shortcomings

The Contribution of Geostatistical Simulations

1. Simulated models represent the actual spatial variability of the deposit which scheduling optimization should use.

2. A group of simulated models quantifies the uncertainty in the description of a mineral deposit that we need to manage suitably while scheduling.
Economic Value, when optimizing, is driven by the economic values of the blocks mined rather than the products produced.

$ \text{VALUE of a BLOCK} = (\text{METAL} \times \text{RECOVERY} \times \text{PRICE} - \text{ORE} \times \text{COSTP}) - \text{ROCK} \times \text{COSTM}$

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**Estimation vs Simulation: Does it Matter?**

*A recall from 1998 (Gold mine in Northern Queensland)*

The expected project NPV has only 2-4% probability to be realized - Testing the conventional plan against simulated scenarios…

[Diagram showing simulated realizations vs forecast from estimated model]

Most probable NPV is A$16.5M, 25% less than the conventional (deterministic) estimate.

Why this? As per the previous grade-tonnage graph, estimation misrepresents volumes of different grade ranges … and more …
**Risk in mine design**

- Why? A major reason is the effect of smoothing

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**Deterministic Workflow**

| Orebody Modelling | Mine Design & Production Scheduling | Financial & Production Forecasts |

**Limitations/shortcomings:**

1. Evaluates the $ value of the block independently of others.
2. Ignores **non-linear transformations** in the processing stream that act on the blend of materials (e.g. non-linear grade-recovery).
   
   **Average in ≠ Average out**

3. Can substantially undervalue the resource by ignoring the power of blending.
4. Ignores uncertainty in material types, chemistry, grades, rock properties.
Some Questions

• Why should we still think that conventional mine planning can provide “optimal” mine plans and production schedules?

• Why should we still think that conventionally optimized Life-of-Mine plans will materialize?

• Why should we still think that we make the best assessments, valuations or forecasts possible?

• Do we really provide the best possible decision support information?

Deterministic Mine Planning

Integer Programming

Objective function

Maximize total $ value

\[(c_1 x_1 + c_2 x_2 + \ldots ) \ldots\]

Subject to constraints

\[a_1 x_1 + a_2 x_2 + \ldots \geq b_1\]

\[\vdots\]

\[a_1 x_p + a_2 x_p + \ldots \geq b_p\]

Estimated Orebody Model

\[C_i = \text{$ value of a block } \ i \]

\[X_i = 1 \text{ if i mined in t, } 0 \text{ otherwise}\]

Period 1

Period p
Stochastic Integer Programming (SIP)

The objective function **now is**

Max $\$ $ value and min expected deviations

\[ (s_{11}x_1^1+s_{21}x_2^1+...\ldots s_{12}x_1^1+s_{22}x_2^1+...) - (\ldots \ldots ) \ldots \]

Subject to

\[ a_{11}x_1^1+a_{21}x_2^1+\ldots = b_1 \quad \text{Simulated model 1} \]
\[ a_{12}x_1^1+a_{22}x_2^1+\ldots = b_1 \quad \text{Simulated model 2} \]
\[ \vdots \]
\[ a_{1r}x_1^r+a_{2r}x_2^r+\ldots = b_r \quad \text{Simulated model r} \]

Stochastic Workflow

Stochastic Orebody Modelling
Simulated Orebody Models

- Sim. 1
- Sim. 2
- Sim. S

A set of simulations describes geological uncertainty and grade variability

Stochastic Mine Design & Production Scheduling
Stochastic Design & Production Schedule

- Year 1
- Year 5

A single mine design and production schedule accounting for and managing risk

Financial & Production Forecasts
Probabilistic Reporting

- Demanded Cohesion
- Conventional method
- Variance

A more realistic forecast of the NPV is obtained than with conventional methods
Oil recovery forecasting (EOR) – Production forecasts:

**Examples**

Forecasts come from multiphase flow simulation

**Estimated reservoir properties**

**Simulated reservoir properties**

**Estimated no longer used in reservoir forecasting**

Simultaneous Stochastic Optimization of Mining Complexes - Mineral Value Chains for Decision Support

Extending models & capitalizing on synergies
Simultaneous Optimization

Mines

Processing streams

Customers & Markets ...

Spot Market

Waste dumps... Tailings...

Rehab ...

One SIP Formulation for the whole Mineral Value Chain

Mining Complexes & Mineral Value Chains

A mining complex may be seen as an integrated business starting from the extraction of materials to a set of sellable products delivered to various customers and/or spot market

Simultaneous optimization of the mining complex/value chain
Economic Block Value, when optimizing, is driven by the economic values of the blocks mined rather than the products produced.

\[
\text{VALUE for A MINING BLOCK} = (\text{METAL} \times \text{RECOVERY} \times \text{PRICE} - \text{ORE} \times \text{COSTP}) - \text{ROCK} \times \text{COSTM}
\]

*CHANGE CONTEXT and USE ONLY geological attributes: Material Types, Grades....*

A mining complex may be seen as an integrated business starting from the extraction of materials to a set of sellable products delivered to various customers and/or spot market.

Simultaneous optimization focuses on the $ value of products sold rather than the $ value of individual blocks and

Generates the optimal cut-off grades.
Simultaneous Optimization

Example
Nickel laterite mineral value chain - Blending policy optimization

*\( T_{\text{max}} \) is the maximum plant feed tonnage

Objectives:
1. Maximize NPV
2. Satisfy SiO\(_2\):MgO blend
3. Minimize deviations from plant capacity target

Simultaneous Optimization

Nickel Laterite Complex – Risk Analysis of Deterministic Design

Orebody simulations quantify:
- Volumetric uncertainty
- Multi-element uncertainty
Simultaneous Optimization

Nickel Laterite Complex – **Deterministic** Simultaneous Optimization

![Graph showing Plant Feed Silica-to-Magnesia Ratio and Plant Feed Tonnage over periods of 36 days with deterministic line highlighted.]

Simultaneous Optimization

Nickel Laterite Complex – Risk Analysis of **Deterministic** Design

![Graph showing Plant Feed Silica-to-Magnesia Ratio and Plant Feed Tonnage over periods of 36 days with P-10, P-90, P-50, and deterministic lines highlighted.]
Simultaneous Stochastic Optimization

Nickel Laterite Mine Production Schedule

Ni Simulations
SiO₂ Simulations
MgO Simulations

Plant Feed SiO₂:MgO
Period

Plant Silica-to-Magnesia Ratio - Stochastic Solution

Plant Feed Tonnage - Stochastic Solution

0% 20% 40% 60% 80% 100% 120% 140% 160% 180% 200%
0 10 20 30 40
Dry Tonnage (% Max. Capacity)
Period (36 days)

P-10 & P-90  P-20 & P-80  P-50  Deterministic

Simultaneous Stochastic Optimization

Nickel Laterite Complex - Simultaneous Stochastic Optimization
Modelling Mining Complexes with Uncertainty

New mathematical models

Simultaneous Stochastic Optimization Formulation

- Adaptable two-stage stochastic integer programming model with CAPEXs:

\[
\max \frac{1}{2} \sum_{t \in T} \sum_{s \in S} \sum_{a \in A} p_{a,t} \cdot v_{a,t,s} - \frac{1}{2} \sum_{t \in T} \sum_{s \in S} \sum_{a \in A} \left( c_{a,t}^+ \cdot u_{a,t,s} + c_{a,t}^- \cdot l_{a,t,s} \right)
\]

Attributes of interest
- Revenues from metal sale
- Mining, processing & stockpiling costs

Penalties for deviations from targets
- Mining, stockpile, processing capacities
- Blending constraints
- Deleterious elements

Change of capacities depends on:
- Quantity purchased \((w_{k,t})\)
- Constraint increase \((k_{a,k})\)
- Life of equipment \((l_k)\)
- Lead time \((\tau_k)\)

\[- \sum_{t \in T} \sum_{k \in K} p_{k,t} \cdot W_{k,t} \]
Simultaneous Stochastic Optimization Formulation

- Adaptable two-stage stochastic integer programming model with CAPEXs:

\[
\max \frac{1}{|S|} \sum_{t \in T} \sum_{s \in S} \sum_{a \in A} p_{a,t} \cdot v_{a,t,s} - \frac{1}{|S|} \sum_{t \in T} \sum_{s \in S} \sum_{a \in A} \left( c_{a,t}^+ \cdot k_{a,t,s} + c_{a,t}^- \cdot l_{a,t,s} \right)
\]

Attributes of interest:
- Revenues from metal sale
- Mining, processing & stockpiling costs
- Penalties for deviations from targets
- Mining, stockpile, processing capacities
- Blending constraints
- Deleterious elements

1. Risk reduction.
2. Risk deferral (geological risk discounting).

Modelling Mining Complexes with Risk Management

Production schedule

<table>
<thead>
<tr>
<th>Sulfides - Mine 1</th>
<th>Sulfides - Mine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal tonnes</td>
<td>Metal tonnes</td>
</tr>
<tr>
<td>Total tonnes</td>
<td>Total tonnes</td>
</tr>
</tbody>
</table>

Destination policies

Processing streams

<table>
<thead>
<tr>
<th>Processing Stream A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total metal</td>
</tr>
<tr>
<td>2. Total tonnes</td>
</tr>
<tr>
<td>3. Head grade</td>
</tr>
<tr>
<td>4. Recovery</td>
</tr>
<tr>
<td>5. Throughput</td>
</tr>
<tr>
<td>6. Metal recovered</td>
</tr>
</tbody>
</table>

Product Value

<table>
<thead>
<tr>
<th>Customer #1 (Contract)</th>
<th>Customer #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Metal</td>
<td>1. Metal</td>
</tr>
<tr>
<td>2. Metal value</td>
<td>2. Metal value</td>
</tr>
</tbody>
</table>

Cash flows are calculated here using products.

No Economic Values for Mining Blocks Used

Decisions, Blending, GEOMET, All topics related to materials mined move here

Uncertainty can be quantified at any stage
Optimization with metaheuristics

- Computationally prohibitive optimization models, IN THE PAST.

Mine 1
400,000 blocks
40 destination decisions/y
30 years
30 simulations

Mine 2
50,000 blocks
40 destination decisions/y
10 years
15 simulations

Mine 3
250,000 blocks
100 destination decisions/y
25 years
20 simulations

- 9,000 joint scenarios
- 18,750,000 scheduling decision variables
- 62,500 destination policy variables
- 540,000 processing stream variables

Optimization with Metaheuristics

Particle Swarm Optimization
Robust destination policies \( d_{j,k} \)
Processing stream variables* \( p_{j,k} \)
Capital expenditures \( w_{k,j} \)

Simulated Annealing
Robust destination policies \( d_{j,k} \)
Robust production schedule* \( s_{j,k} \)
Capital expenditures \( w_{k,j} \)

But also TS, VNS, LNS, and other hybrids ...
Other challenges

• Metaheuristics are not a cure-all …
  • Which parameters have a significant impact on the algorithm performance and how can they be adjusted?
  • Which metaheuristic will be the most efficient for optimizing the mining complex at hand?
  • Significant programming effort to adapt them to new mining complexes

• Can we carry out the choices of parameters and/or of (meta)heuristics in an automatic way?

Hyper-heuristics

“A hyper-heuristic is a search method or a learning mechanism for selecting or generating heuristics to solve computational search problems”, Burke et al. 2013

A heuristic to find the best heuristic for a given situation …
What is different?

Traditional search techniques

- Operate on the search space of solutions

Hyper-heuristics

- Use a score-based learning mechanism, whereby a score is associated with each heuristic reflecting its past performance
- Select based on these scores

Potential solutions

Potential solutions

Numerical results
Overview of the benchmark datasets

- 43 instances: one mine

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of instances</td>
<td>8</td>
<td>2</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td># of blocks</td>
<td>[4273 ; 34,981]</td>
<td>[26,021; 40,762]</td>
<td>[4273; 40,762]</td>
<td>[21,966; 22,720]</td>
<td>40,090</td>
<td>[14,118; 48,821]</td>
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<tr>
<td># of periods</td>
<td>[3; 10]</td>
<td>[11; 13]</td>
<td>[3; 13]</td>
<td>[11; 12]</td>
<td>21</td>
<td>[6; 16]</td>
</tr>
<tr>
<td># of scenarios</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>2025</td>
</tr>
<tr>
<td># of processors</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td># of stockpiles</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Metal Type</td>
<td>Cu and Au</td>
<td>Cu and Au</td>
<td>Cu and Au</td>
<td>Cu</td>
<td>Cu</td>
<td>Cu and Au</td>
</tr>
<tr>
<td># Binary var.</td>
<td>[12,819; 314,829]</td>
<td>[338,273; 448,382]</td>
<td>[12,819; 448,382]</td>
<td>[241,615; 272,640]</td>
<td>841,890</td>
<td>[84,708; 683,494]</td>
</tr>
<tr>
<td># Continuous var.</td>
<td>[240; 800]</td>
<td>[880; 1040]</td>
<td>[180; 780]</td>
<td>[1320; 1420]</td>
<td>2520</td>
<td>[900; 1920]</td>
</tr>
</tbody>
</table>

**CPU CPLEX 12.2 (LR) > 4 WEEKS**
Otherwise, in [0.23 min ; 11 days]

Benchmarking ...

**HH#**

**TS for L1 and L2**

**DLS = Hybrid VND and NFA for S1-S4**
Benchmarking …

An Operating Gold Mining Complex

Twin Creeks Gold Mining Complex, Nevada
Gold Mining Complex

Stockpiles

Pit 1

Extraction Capacity

Other Sources

A

B

Autoclave

Waste Dumps

Leach

Mill

Stockpiles

Blending is crucial!
(SS/CO₃ => acid to reduce carbonate concentration)

Base Case - Sources of Supply Uncertainty

Pit 1

Stochastic simulations

Simulated Sulphide Stockpile

Other Sources

A

B

Autoclave

Mill

Leach

Stochastic simulations
Base Case - Gold Recovery & Risk Analysis

Cumulative gold recovered – First 6 years

Year

Million Oz

2013 2014 2015 2016 2017 2018

6%

Base Case forecast  P10  P50  P90

Base Case - DCF & Risk analysis

Cumulative DCF – First 6 years

Year

0% 10% 20% 30% 40% 50% 60%

2013 2014 2015 2016 2017 2018

9%

Base Case forecast  P10  P50  P90
**Base Case - Blending: SS and Acid**

- Sulfide sulfur is not a major problem
- Carbonate materials demand excessive amounts of acid and above legal limits

**Twin Creeks Gold Mining Complex, Nevada**

**Stochastic** Production Schedule

Fitted to - Constrained by the Existing Pit Designs
Modified Stochastic Schedule - Vista Pit

Stochastic vs conventional schedules:
Substantially different parts of the pit are mined at the same year

Modified Stochastic Schedule - Mega Pit

Stochastic vs conventional schedules:
Substantially different parts of the pit are mined at the same year
Modified (practical) Stochastic Schedule

Cumulative Gold Recovered - LOM

Practical Stochastic Schedule

Cumulative Gold Recovered – First 6 years
- Sulfide sulfur is well controlled
- Acid requirement is below the maximum consumption allowed in the long-term plan (and legal regulations)
Practical Stochastic Schedule

Cumulative DCF – First 6 years

Year

Practical Stochastic Schedule

What if the stochastic scheduler finds a different and larger ultimate pit?
Meaning: If this approach was applied from the start, ie several years earlier, there would be more gold produced from the same assets.
ALL IMPROVEMENTS ARE DUE TO:

Managing Technical Risk from materials mined

*and at the same time*

Capitalizing on the Synergies between all parts of the mining complex

The End

Thanks are in order to our

COSMO Industry Members

And Funding Agencies