Simultaneous Stochastic Optimization of Mining Complexes / Mineral Value Chains

A review of applications, solution methods and key findings

Amina Lamghari
Roussos Dimitrakopoulos

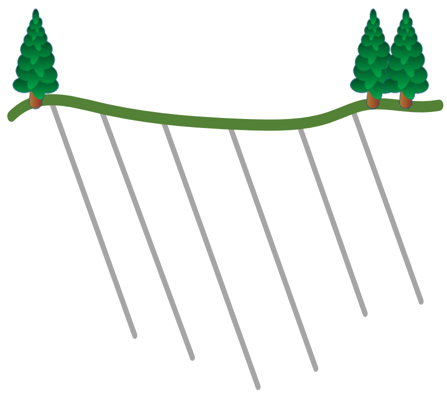
COSMO *Stochastic Mine Planning Laboratory* - http://cosmo.mcgill.ca/

Introduction & Basics

Conventional long-term planning
Orebody models
Major limitations
Stochastic workflow

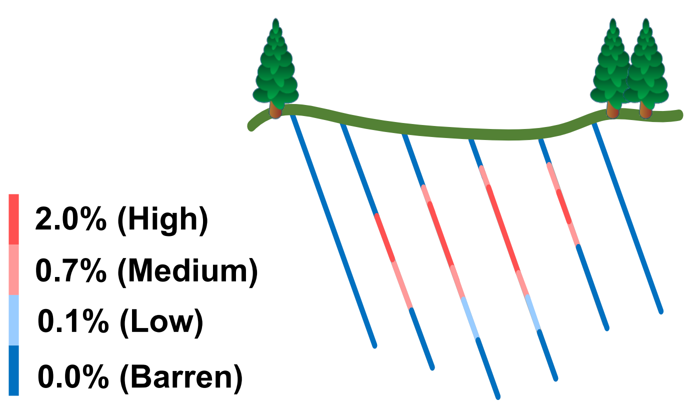
Quick Introduction to Mining

- From small holes to big pits: **drill** for a new deposit.



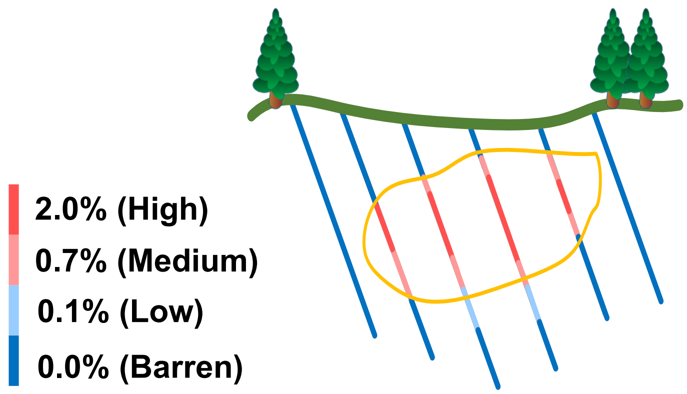
Quick Introduction to Mining

- From small holes to big pits: **assay** the drill cores ("samples") for metal content ("**grade**", %, gpt, ppm)



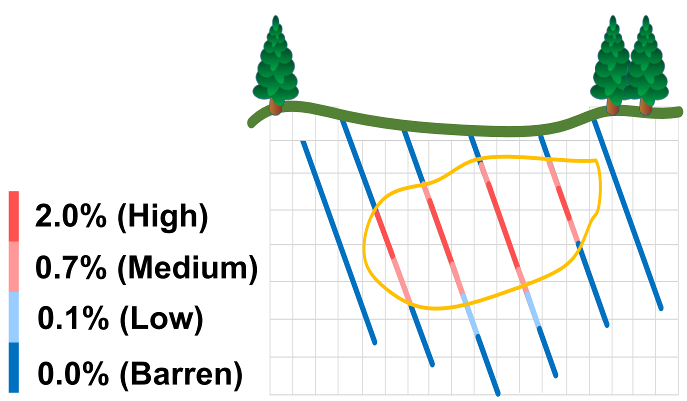
Quick Introduction to Mining

- From small holes to big pits: define the mineralized volume ("**orebody**").



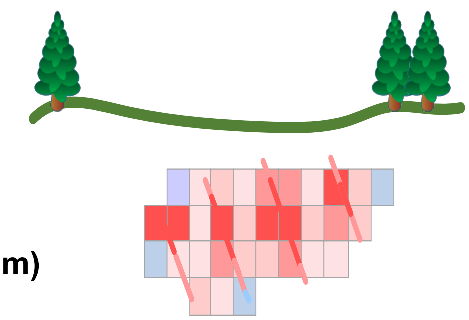
Quick Introduction to Mining

- From small holes to big pits: **discretize** the model into 3D volumes (often, but not necessarily, "**blocks**").



Quick Introduction to Mining

- From small holes to big pits: create **models** for the metal content (grades) for each of the volumes (blocks) in the orebody ("**geostatistics**").

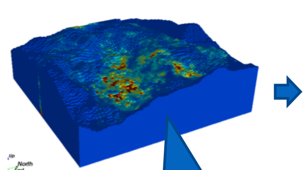


2.0% (High)
0.7% (Medium)
0.1% (Low)
0.0% (Barren)

Conventional / Deterministic Workflow

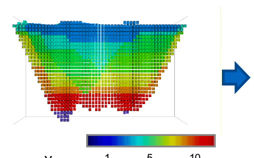
Orebody Modelling → Mine Design & Production Scheduling → Financial & Production Forecasts

Estimated Orebody Model



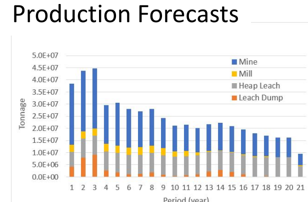
Can a single estimated model represent a mineral deposit? (Grade variability, uncertainty)

Deterministic Design



Is this design the optimal / 'best'?

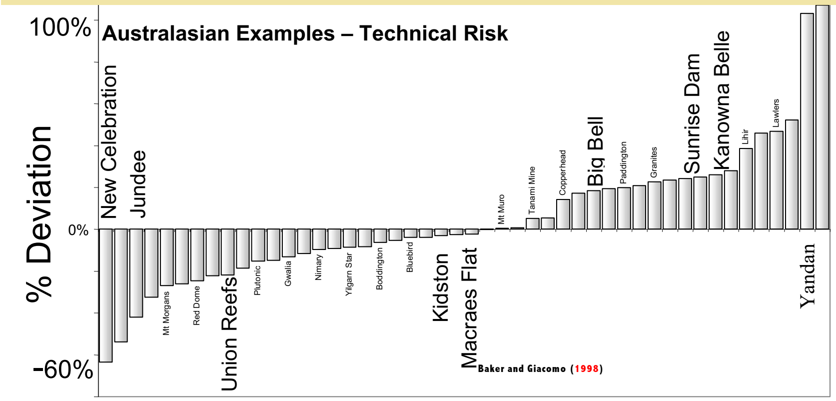
Production Forecasts



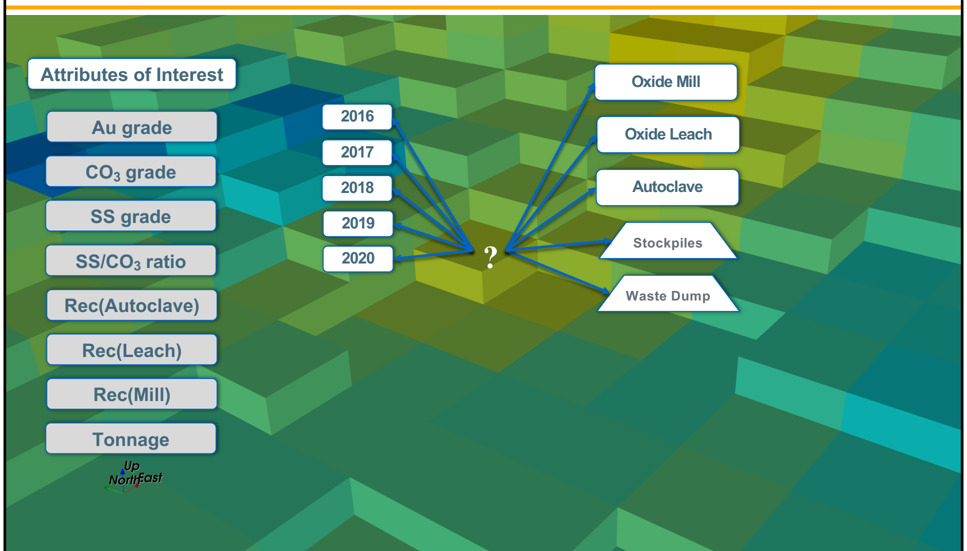
Are we able to meet expected forecasts?

Deterministic Workflow

~80% of Failures Due to Geological Risk



Mining Decisions



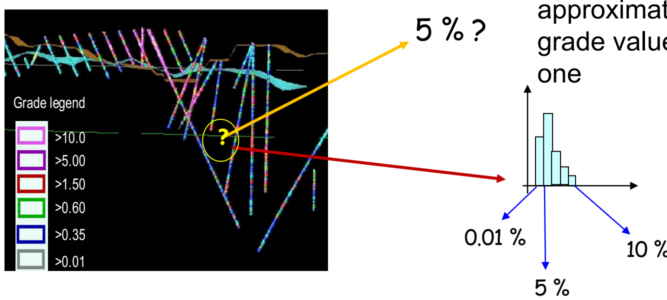
Traditional production scheduling methodologies neglect uncertainty and variability!

Source: M.Godoy, Newmont Gold, SME 2016

Traditional Orebody Models: Some Limitations and *Shortcomings*

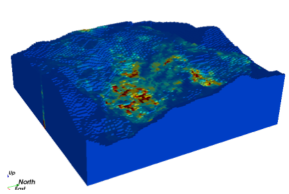
Conventional models **DO NOT account for uncertainty….**

Estimation methods try to approximate some average grade value ... not the actual one



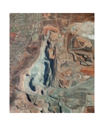
Estimation vs Simulation

- **Estimated Orebody Model**

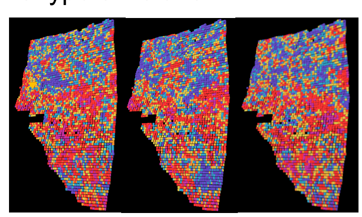


Quantifying Uncertainty

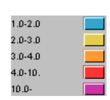
- **Simulated Orebody Models.** This is a Monte Carlo type simulation …



A mature, well drilled and understood gold deposit



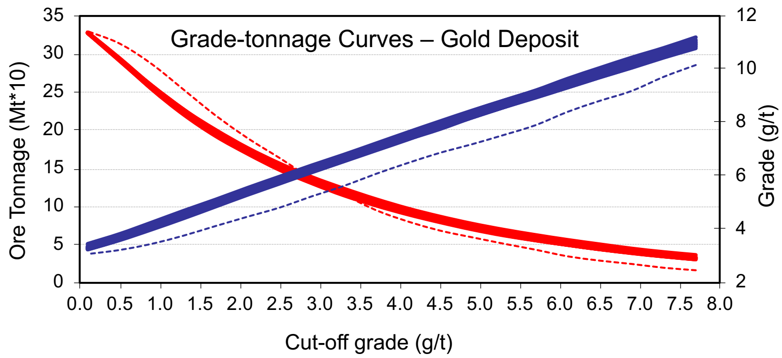
3 simulated scenarios of the same section (SMU grade)



Traditional Orebody Models - Limitations & *Shortcomings*

Simulated grades

Estimated (- - -, - - -) *vs simulated* models (— , —) as inputs to …



The representation of a mineral deposit and related pertinent attributes – estimated vs simulated - MATTERS …

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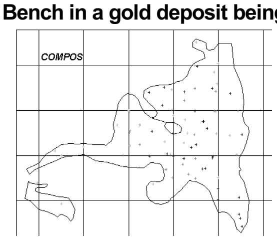
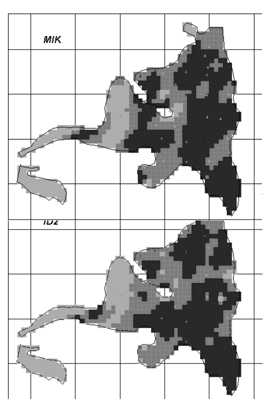
Traditional Orebody Models - Limitations & *Shortcomings*

Bench in a gold deposit being mined



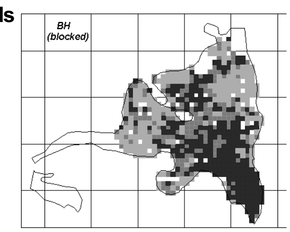
Black indicates DDH grade above 1.3 g/t and grey between 0.7 and 1.3 g/t

Estimated deposit bench, 2 methods



10x10x5m blocks

Real blast hole data



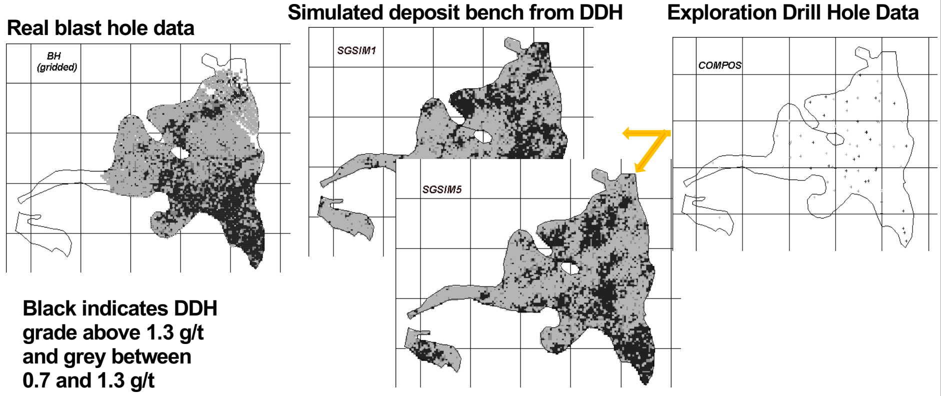
Real mineral deposits are highly variable, not smooth

Traditional Orebody Models - Limitations & *Shortcomings*

Real blast hole data

Simulated deposit bench from DDH

Exploration Drill Hole Data



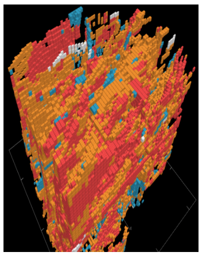
Black indicates DDH grade above 1.3 g/t and grey between 0.7 and 1.3 g/t

Traditional Orebody Models - Limitations & *Shortcomings*

The Contribution of Geostatistical Simulations

1. **Simulated models represent the actual *spatial variability* of the deposit which scheduling optimization should use**
2. **A group of simulated models *quantifies the uncertainty* in the description of a mineral deposit that we *need to manage* suitably while scheduling**

Mine Planning



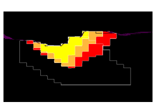
Economic Value, when optimizing, is driven by the economic values of the blocks mined rather than the products produced.

$ VALUE of a BLOCK =

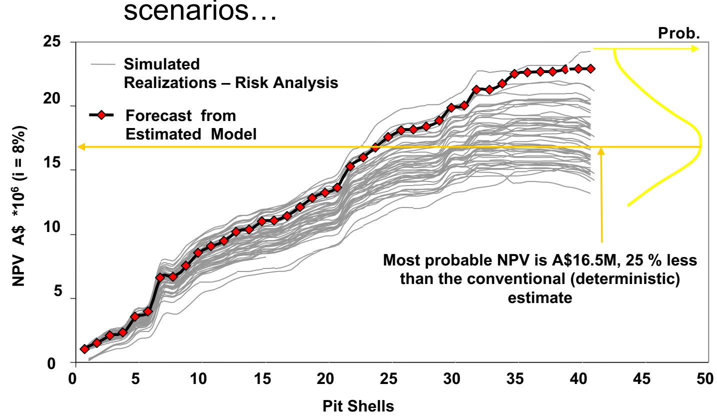
(METAL*RECOVERY*PRICE - ORE*COSTP)

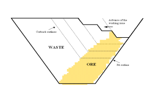
- ROCK*COSTM

Estimation vs Simulation: Does it Matter?



A recall from 1998 (Gold mine in Northern Queensland)
The expected project NPV has only 2-4% probability to be realized - Testing the conventional plan against simulated scenarios…



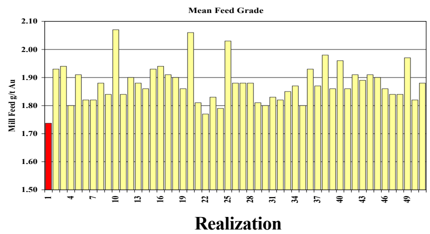


Why this? As per the previous grade-tonnage graph, estimation misrepresents volumes of different grade ranges … and more …

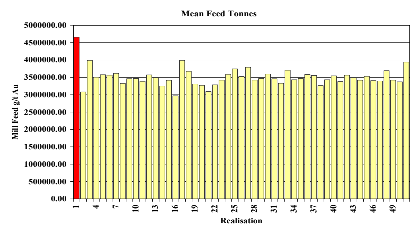
Risk in mine design

- Why? A major reason is the effect of smoothing

Average Grade of Mill Feed Ore over the Life of Mine



Average Ore Tonnes of Mill Feed Ore over the Life of Mine



Deterministic Workflow

Orebody Modelling → Mine Design & Production Scheduling → Financial & Production Forecasts

Limitations/shortcomings:

1. Evaluates the $ value of the block independently of others.
2. Ignores ***non-linear transformations*** in the processing stream that act on the blend of materials (e.g. non-linear grade-recovery).

   Average in ≠ Average out

3. Can substantially undervalue the resource by ignoring the power of blending.
4. Ignores uncertainty in material types, chemistry, grades, rock properties.

Some Questions

- Why should we still think that conventional mine planning can provide "optimal" mine plans and production schedules?
- Why should we still think that conventionally optimi zed Life-of-Mine plans will materialize?
- Why should we still think that we make the best assessments, valuations or forecasts possible?
- Do we really provide the best possible decision support information?

Deterministic Mine Planning

Integer Programming

Objective function

Maximize total $ value

$(c_1x_1^1+c_2x_2^1+\ldots.)\ \ldots$

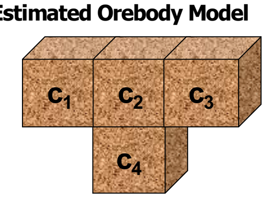
Subject to constraints

$$a_1x_1^1+a_2x_2^1+\ldots. \geq b_1 \quad \rightarrow \textbf{Period 1}$$

$$\vdots$$

$$a_1x_1^p+a_2x_2^p+\ldots. \geq b_p \quad \rightarrow \textbf{Period p}$$

Estimated Orebody Model



c_i = $ value of a block i

X_i^t = 1 if i mined in t, 0 otherwise

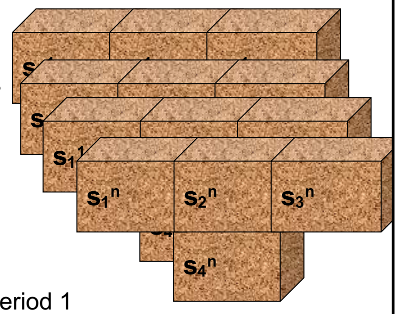
Stochastic Mine Planning (start)

Stochastic Integer Programming (SIP)

The objective function **now is**

Max $ value and min expected deviations

$$(s_{11}x_1^1+s_{21}x_2^1+\ldots$$
$$s_{12}x_1^1+s_{22}x_2^1+\ldots) - (\ \ldots\ )\ \ldots$$



Subject to

$$\left.\begin{aligned} a_{11}x_1^1+a_{21}x_2^1+\ldots &= b_1 \\ a_{12}x_1^1+a_{22}x_2^1+\ldots &= b_1 \\ &\vdots \\ a_{1r}x_1^p+a_{2r}x_2^p+\ldots &= b_p \end{aligned}\right\}$$

Period 1
Simulated model 1

Simulated model 2

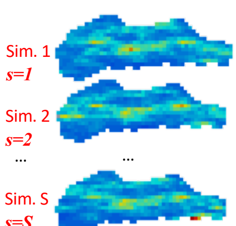
Period p

Simulated model r

Stochastic Workflow

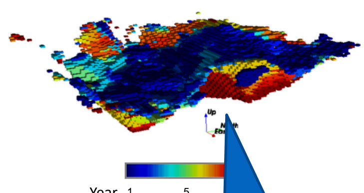
Stochastic Orebody Modelling → Stochastic Mine Design & Production Scheduling → Financial & Production Forecasts

Simulated Orebody Models



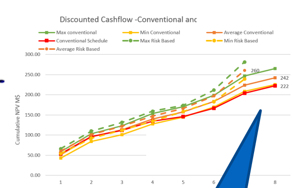
A set of simulations describes geological uncertainty and grade variability

Stochastic Design & Production Schedule



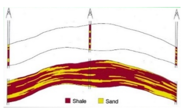
A single mine design and production schedule accounting for and managing risk

Probabilistic Reporting



A more realistic forecast of the NPV is obtained than with conventional methods

Cross-disciplinary Learning

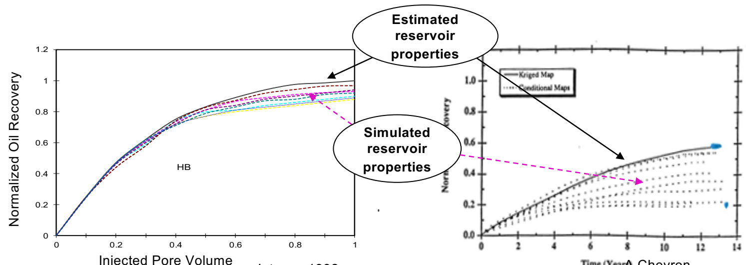


Other fields of Engineering: Industry practice in Petroleum Reservoir Engineering has moved away from estimation models since the late 1980's (stemming from the Stanford University related research - Prof. A. Journel)

Oil recovery forecasting (EOR) – Production forecasts:

Examples

Forecasts come from multiphase flow simulation



Intevep 1992

A Chevron example-1990

Estimation no longer used in reservoir forecasting

Simultaneous Stochastic Optimization of Mining Complexes - Mineral Value Chains for Decision Support

Extending models & capitalizing on synergies

Simultaneous Optimization

Mines

Processing streams

Customers & Markets …

Spot Market

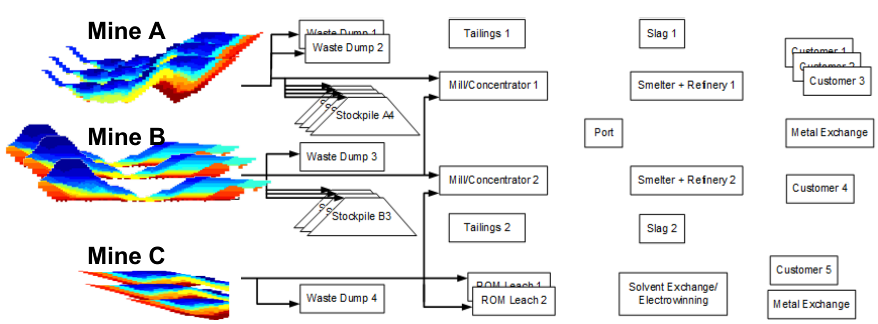
Waste dumps…. Tailings… Rehab …

One SIP Formulation for the whole Mineral Value Chain

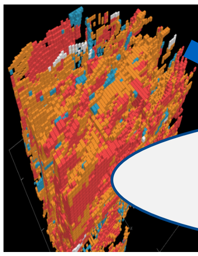
Mining Complexes & Mineral Value Chains

A mining complex may be seen as an *integrated business* starting from the extraction of materials to a set of sellable products delivered to various customers and/or spot market

Simultaneous optimization of the mining complex/value chain



Mining Complexes & Mineral Value Chains



Economic Block Value, when optimizing, is driven by the economic values of the blocks mined rather than the products produced.

CHANGE CONTEXT and USE ONLY geological attributes: Material Types, Grades ….

(METAL*RECOVERY*PRICE - ORE*COSTP)

- ROCK*COSTM

Mining Complexes & Mineral Value Chains

A mining complex may be seen as an *integrated business* starting from the extraction of materials to a set of sellable products delivered to various customers and/or spot market

Simultaneous optimization focuses on the

$ value of products sold

rather than the

$ value of individual blocks

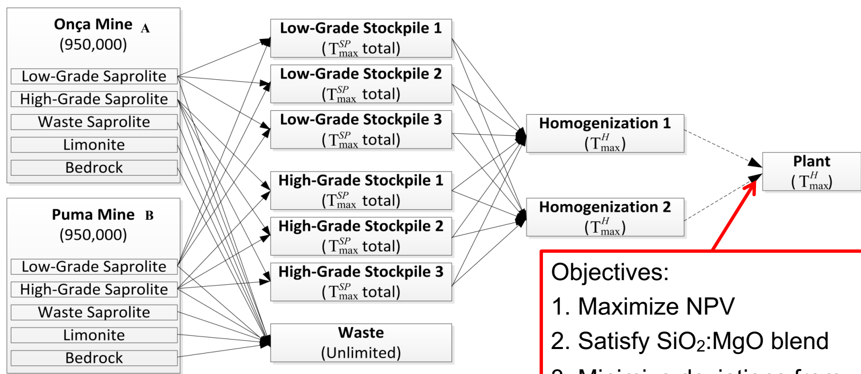
and

Generates the optimal cut-off grades

Simultaneous Optimization

Example

Nickel laterite mineral value chain - Blending policy optimization



Objectives:

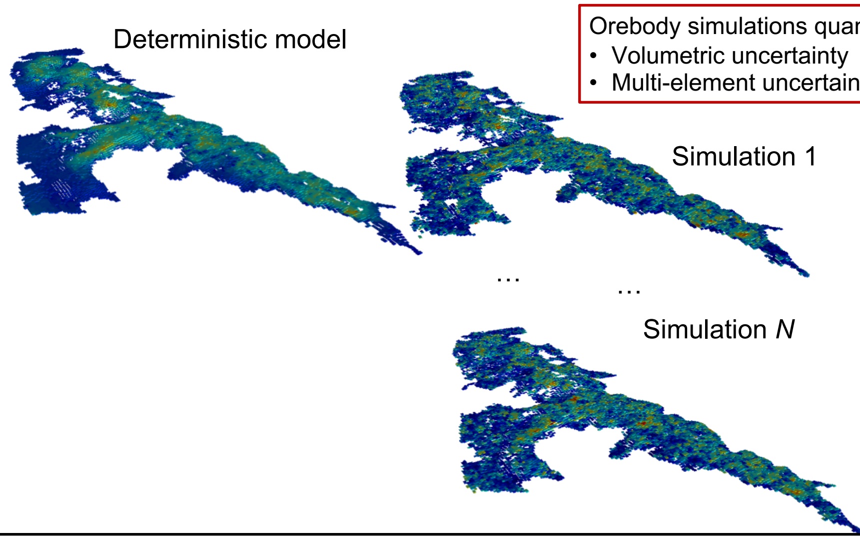
1. Maximize NPV
2. Satisfy SiO_2:MgO blend
3. Minimize deviations from plant capacity target

*T_{max} is the maximum plant feed tonnage

Simultaneous Optimization

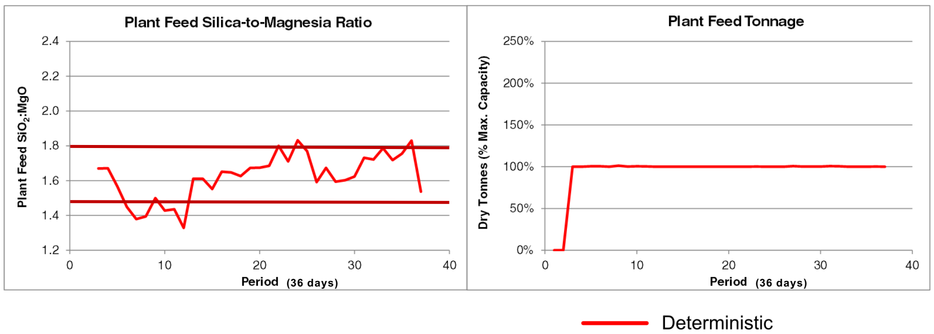
Nickel Laterite Complex – Risk Analysis of Deterministic Design

Orebody simulations quantify:
- Volumetric uncertainty
- Multi-element uncertainty



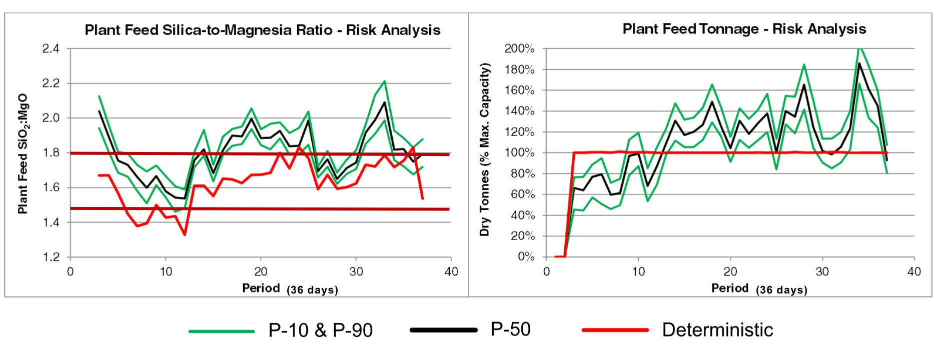
Simultaneous Optimization

Nickel Laterite Complex – Deterministic Simultaneous Optimization

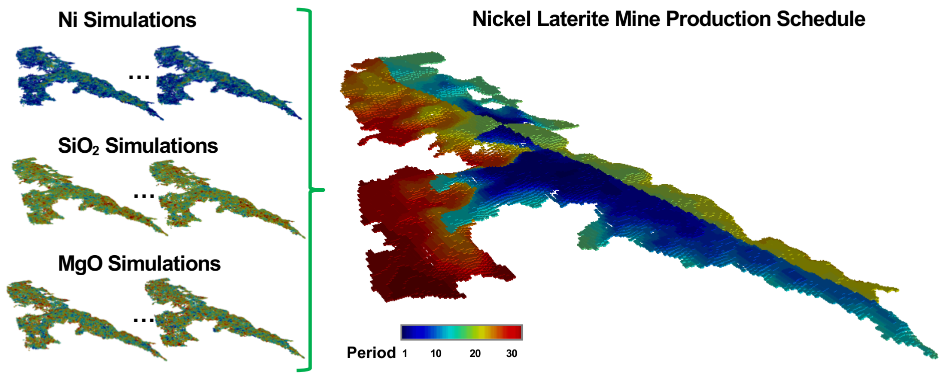


Simultaneous Optimization

Nickel Laterite Complex – Risk Analysis of Deterministic Design

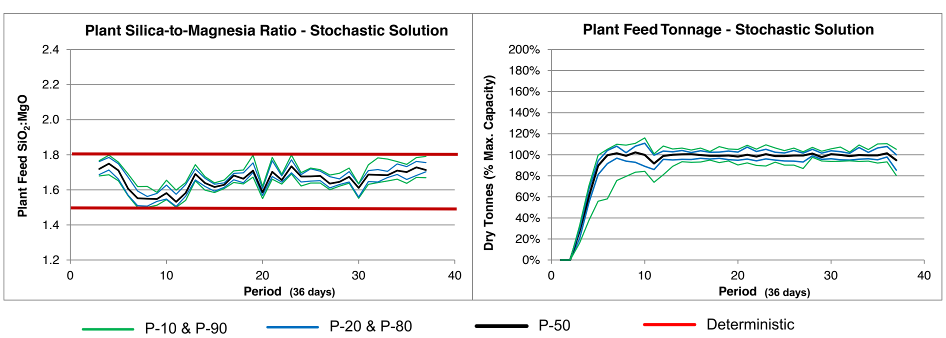


Simultaneous Stochastic Optimization



Simultaneous Stochastic Optimization

Nickel Laterite Complex - Simultaneous Stochastic Optimization



Modelling Mining Complexes with Uncertainty

New mathematical models

Simultaneous Stochastic Optimization Formulation

- Adaptable two-stage stochastic integer programming model with CAPEXs:

$$\max \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{a\in\mathbb{A}} p_{a,t}\cdot v_{a,t,s}}_{} - \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{a\in\mathbb{A}} \left(c^{+}_{a,t}\cdot u_{a,t,s} + c^{-}_{a,t}\cdot l_{a,t,s}\right)}_{}$$

Attributes of interest
- Revenues from metal sale
- Mining, processing & stockpiling costs

Penalties for deviations from targets
- Mining, stockpile, processing capacities
- Blending constraints
- Deleterious elements

$$\underbrace{-\sum_{t\in\mathbb{T}}\sum_{k\in\mathbb{K}} p_{k,t}\cdot w_{k,t}}_{\text{CAPEX}}$$

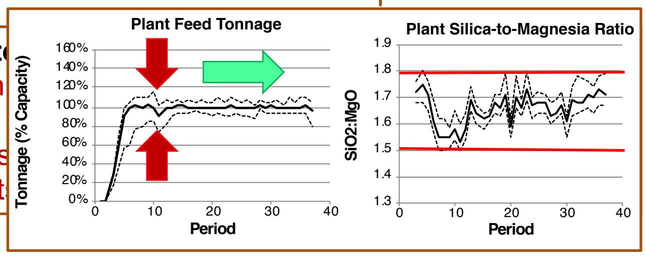
Change of capacities depends on:
- Quantity purchased ($w_{k,t'}$)
- Constraint increase ($\kappa_{a,k}$)
- Life of equipment (λ_k)
- Lead time (τ_k)

Simultaneous Stochastic Optimization Formulation

- Adaptable two-stage stochastic integer programming model with CAPEXs:

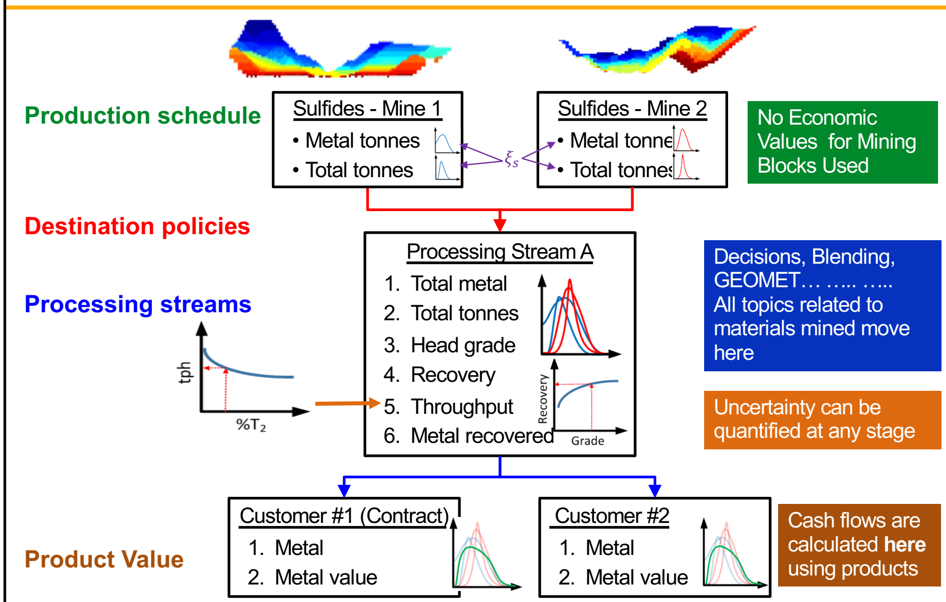
$$\max \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{a\in\mathbb{A}} p_{a,t}\cdot v_{a,t,s}}_{} - \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{a\in\mathbb{A}}\left(\boxed{c_{a,t}^{+}}\cdot\boxed{u_{a,t,s}}+\boxed{c_{a,t}^{-}}\cdot\boxed{l_{a,t,s}}\right)}_{}$$

Attributes of int
- Revenues from metal sale
- Mining, process stockpiling cost



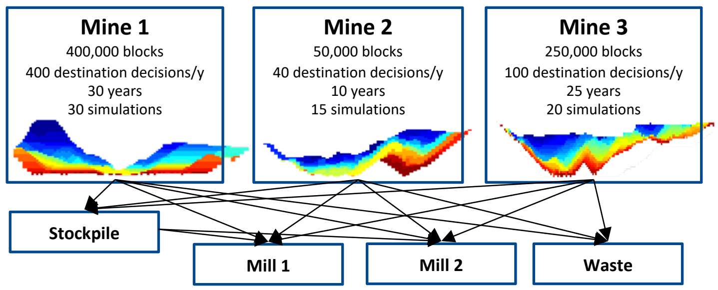
1. **Risk reduction.**
2. **Risk deferral (geological risk discounting).**

Modelling Mining Complexes with Risk Management



Optimization with metaheuristics

- Computationally prohibitive optimization models, IN THE PAST.



- **9,000 joint scenarios**
- **18,750,000 scheduling decision variables**
- **62,500 destination policy variables**
- **540,000 processing stream variables**

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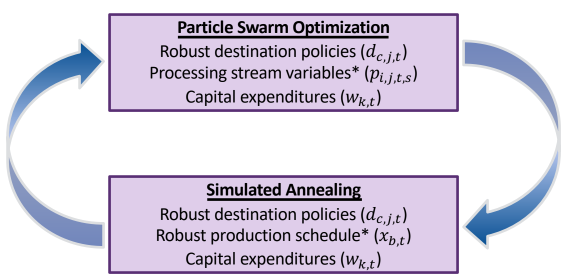
Optimization with Metaheuristics



But also TS, VNS, LNS, and other hybrids …

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Other challenges

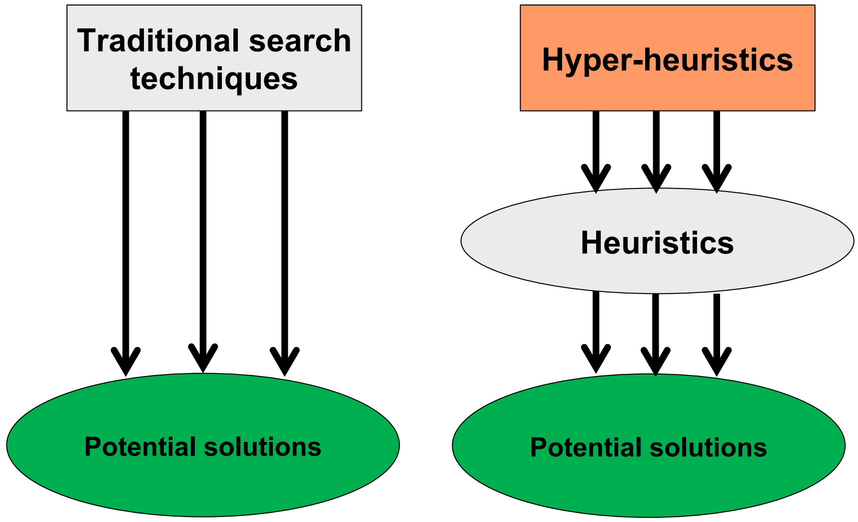
- Metaheuristics are not a cure-all …
  - Which parameters have a significant impact on the algorithm performance and how can they be adjusted?
  - Which metaheuristic will be the most efficient for optimizing the mining complex at hand?
  - Significant programming effort to adapt them to new mining complexes
- Can we carry out the choices of parameters and/or of (meta)heuristics in an automatic way?

Hyper-heuristics

"A hyper-heuristic is a search method or a learning mechanism for selecting or generating heuristics to solve computational search problems", Burke et al. 2013

A heuristic to find the best heuristic for a given situation …

What is different?



- Operate on the search space of solutions

- Use a score-based learning mechanism, whereby a score is associated with each heuristic reflecting its past performance
- Select based on these scores

Numerical results

Overview of the benchmark datasets

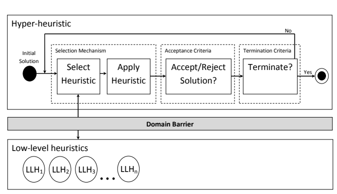
- 43 instances: one mine

	L1	L2	S1	S2	S3	S4
# of instances	8	2	10	10	10	3
# of blocks	[4273 ; 34,981]	[26,021 ; 40,762]	[4273 ; 40,762]	[21,965 ; 22,720]	40,090	[14,118 ; 48,821]
# of periods	[3 ; 10]	[11 ; 13]	[3 ; 13]	[11 ; 12]	21	[6 ; 16]
# of scenarios	20	20	20	20	20	20/25
# of processors	1	1	1	2	2	2
# of stockpiles	0	0	1	2	2	2
Metal Type	Cu and Au	Cu and Au	Cu and Au	Cu	Cu	Cu and Au
# Binary var.	[12,819 ; 314,829]	[338,273 ; 448,382]	[12,819 ; 448,382]	[241,615 ; 272,640]	841,890	[84,708 ; 683,494]
# Continous var.	[240 ; 800]	[880 ; 1040]	[180 ; 780]	[1320 ; 1420]	2520	[900 ; 1920]

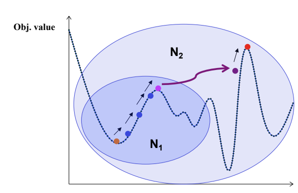
CPU CPLEX 12.2 (LR) > 4 WEEKS

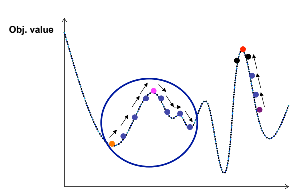
Otherwise, in [0.23 min ; 11 days]

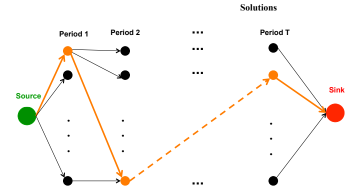
Benchmarking …



HH#



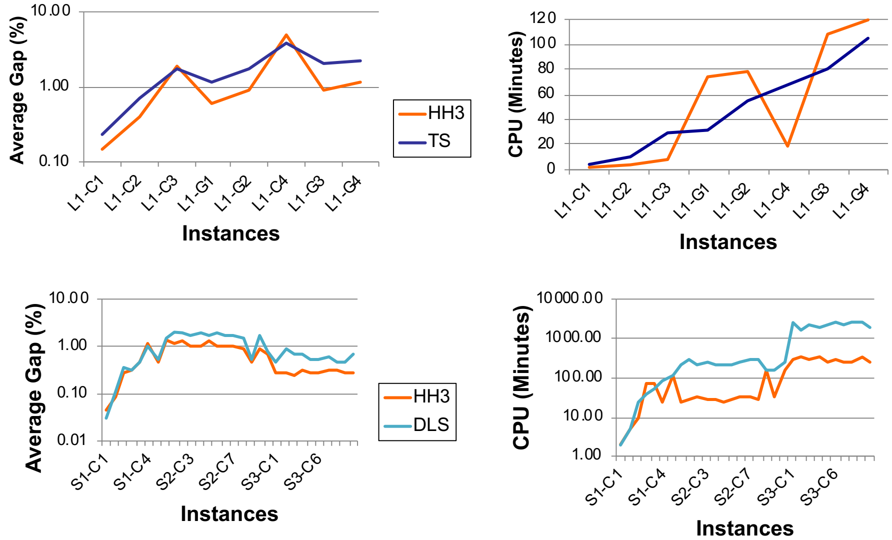




TS for L1 and L2

DLS = Hybrid VND and NFA for S1-S4

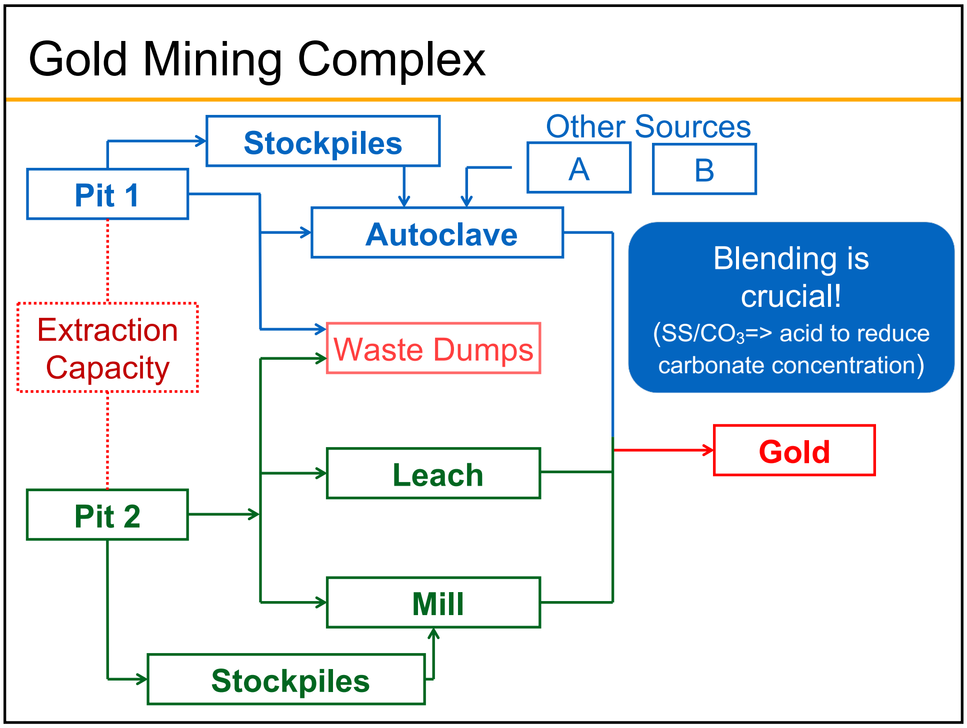
Benchmarking …

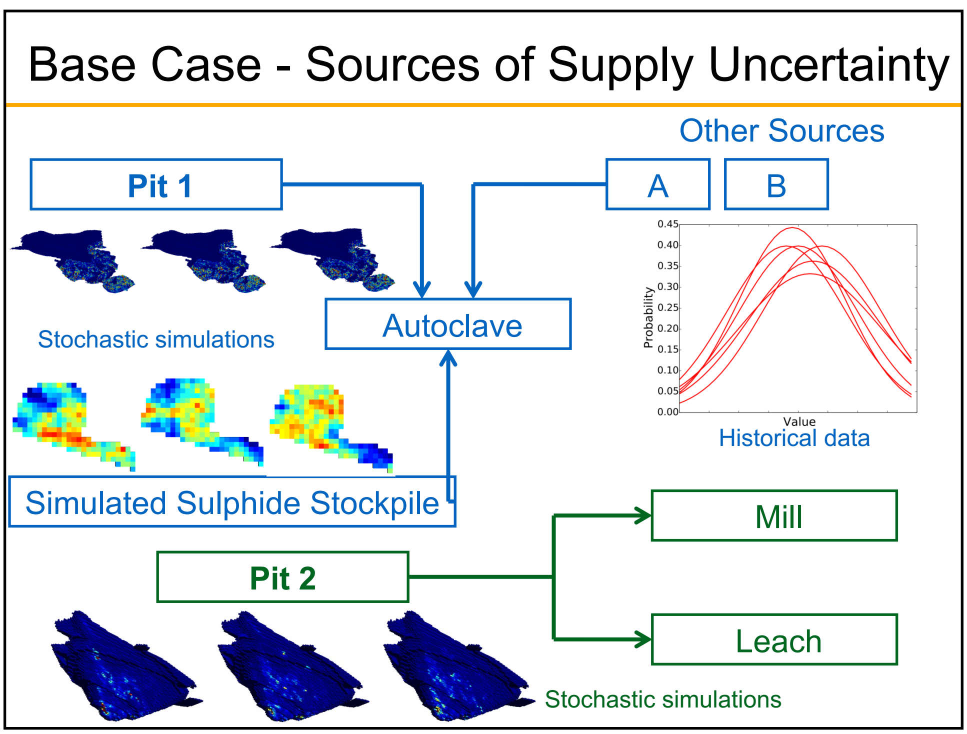


Twin Creeks Gold Mining Complex, Nevada

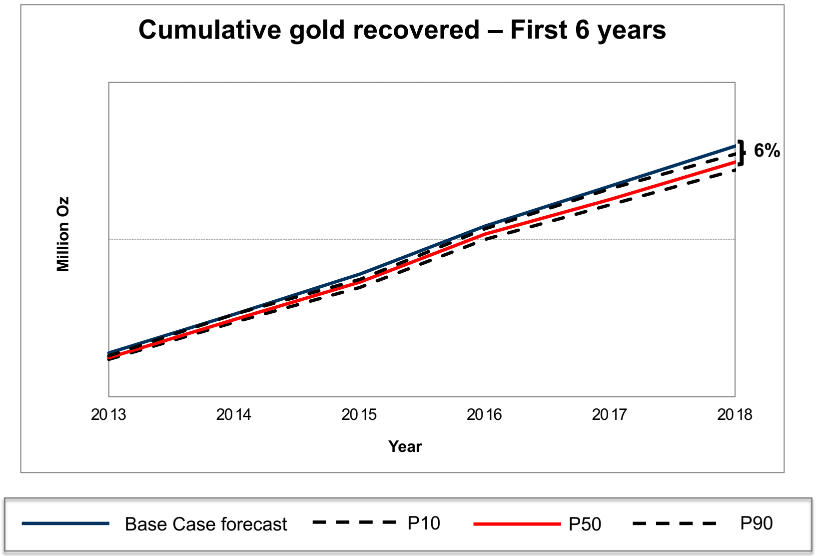
An Operating

Gold Mining Complex

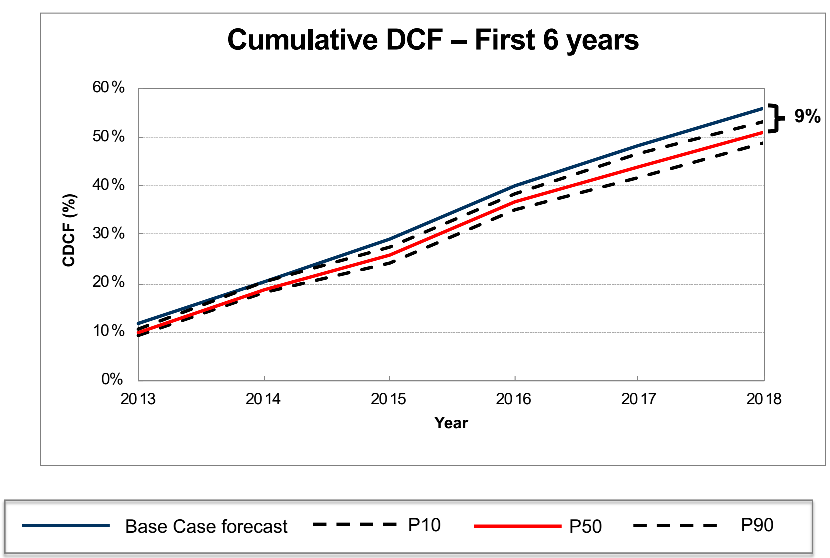

Gold Mining Complex
Stockpiles
Other Sources
A
B
Pit 1
Autoclave
Blending is crucial!
(SS/CO3=> acid to reduce carbonate concentration)
Extraction Capacity
Waste Dumps
Gold
Leach
Pit 2
Mill
Stockpiles


Base Case - Sources of Supply Uncertainty
Other Sources
Pit 1
A
B
Stochastic simulations
Autoclave
Probability
Value
Historical data
Simulated Sulphide Stockpile
Mill
Pit 2
Leach
Stochastic simulations

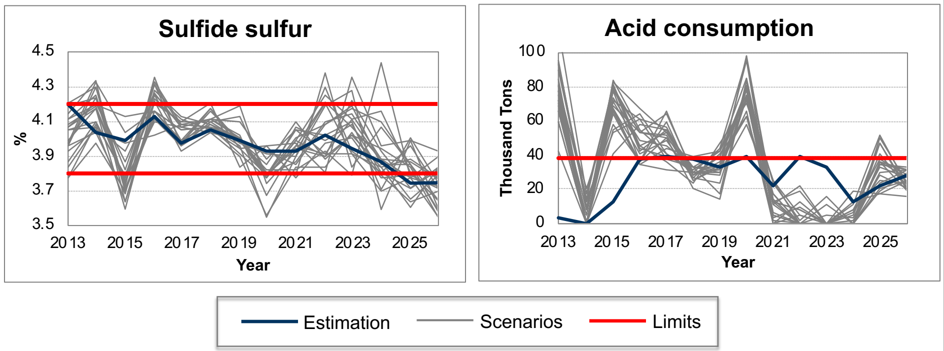
Base Case - Gold Recovery & Risk Analysis



Base Case - DCF & Risk analysis



Base Case - Blending: SS and Acid



- Sulfide sulfur is not a major problem
- Carbonate materials demand excessive amounts of acid and above legal limits

Twin Creeks Gold Mining Complex, Nevada

Stochastic Production Schedule

Fitted to - Constrained

by the

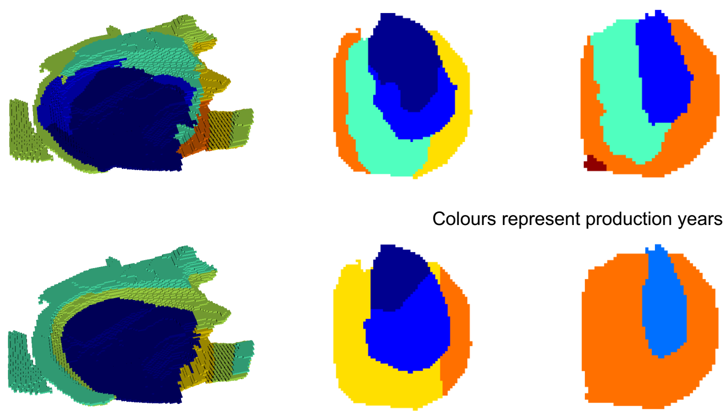
Existing Pit Designs

Modified Stochastic Schedule - Vista Pit

Full View **Bench 0500** **Bench 0360**

Practical Stochastic plan

Base case plan



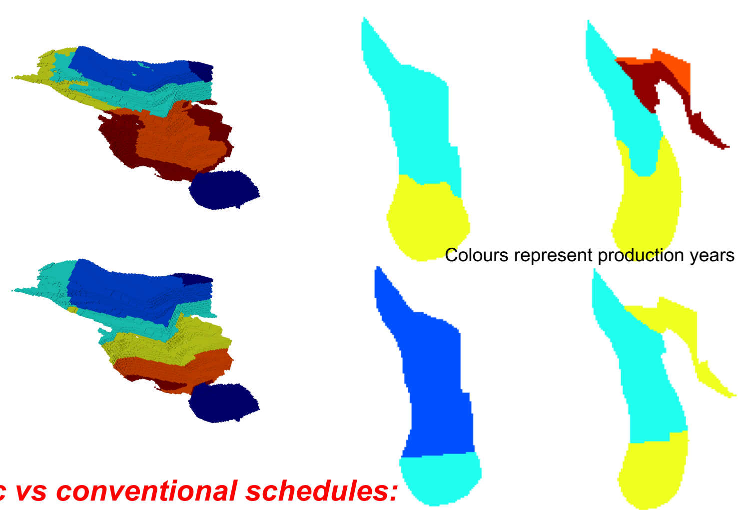
Stochastic vs conventional schedules:
Substantially different parts of the pit are mined at the same year

Modified Stochastic Schedule - Mega Pit

Full View **Bench 3400** **Bench 2460**

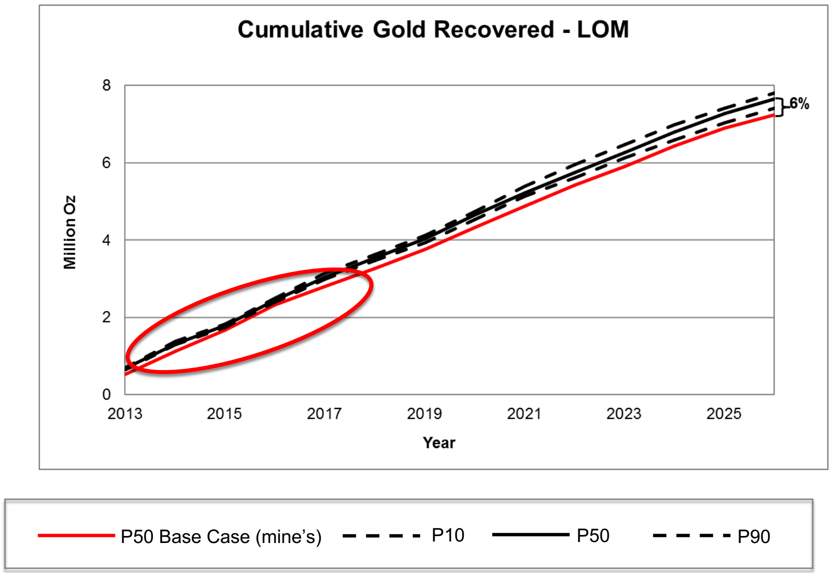
Practical stochastic plan

Base case plan

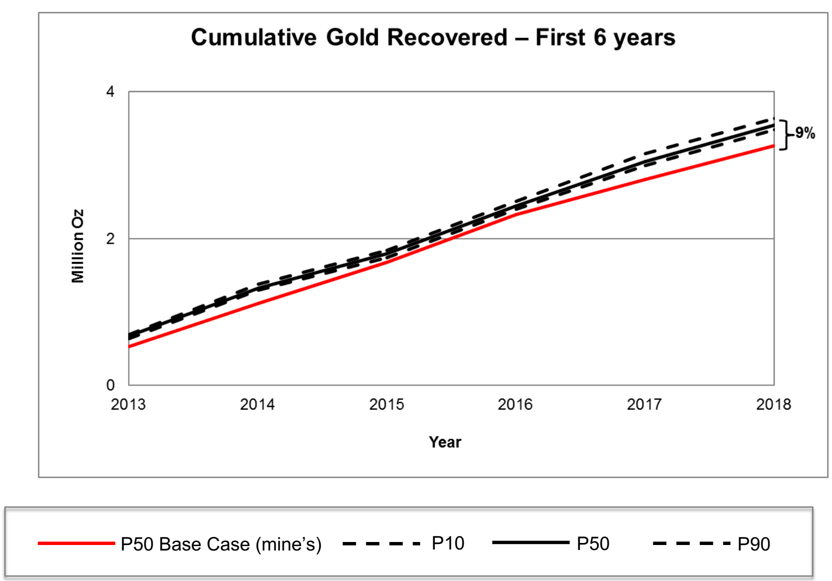


Stochastic vs conventional schedules:
Substantially different parts of the pit are mined at the same year

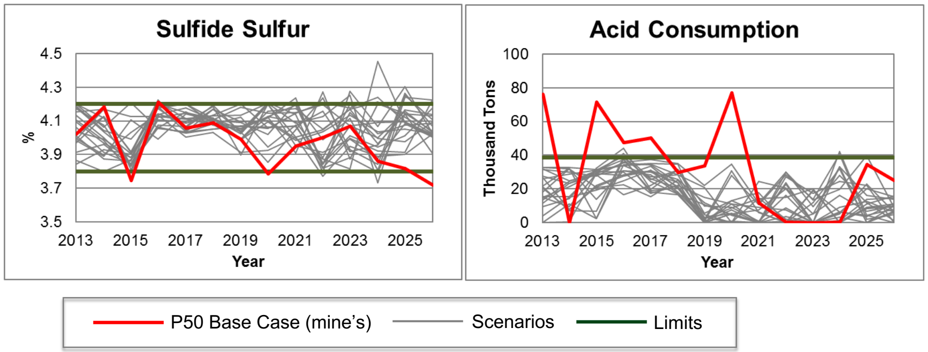
Modified (practical) Stochastic Schedule



Practical Stochastic Schedule

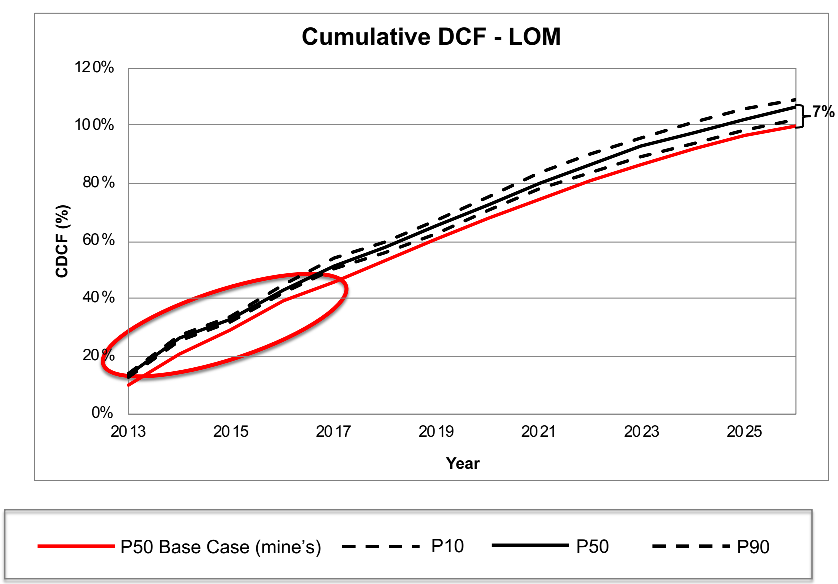


Practical Stochastic Schedule

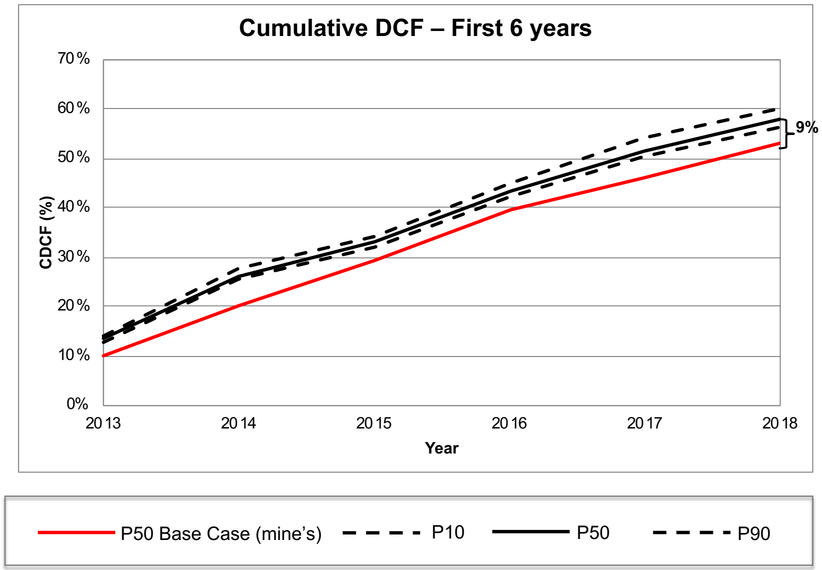


- Sulfide sulfur is well controlled
- Acid requirement is below the maximum consumption allowed in the long-term plan (and legal regulations)

Modified (practical) Stochastic Schedule



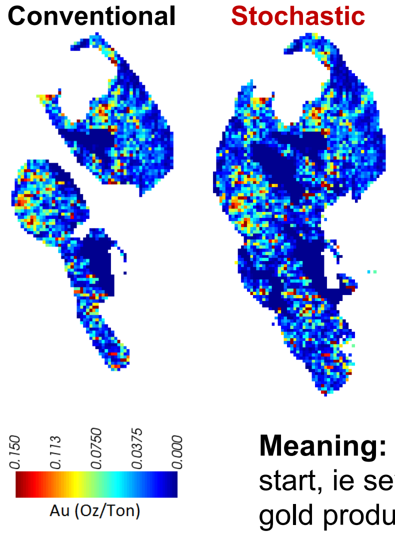
Practical Stochastic Schedule



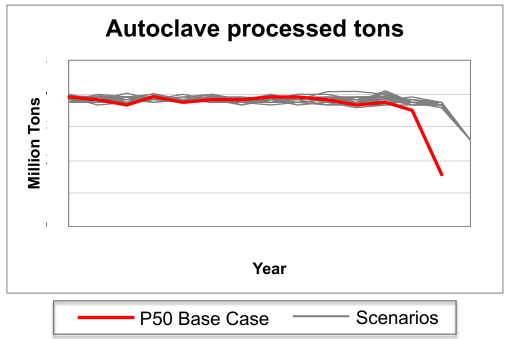
What if the stochastic scheduler finds a different and larger ultimate pit?

Stochastic schedule - More ore, larger pit

Pit A – Bench X

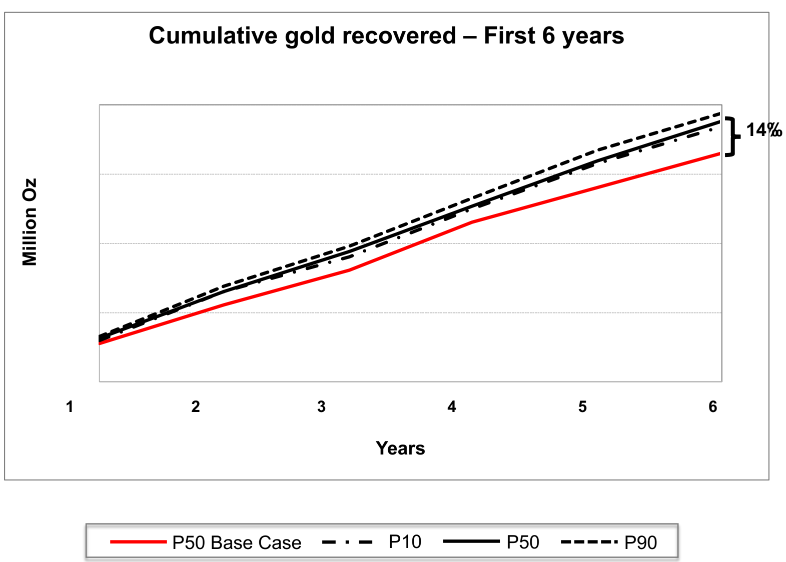


1 extra year of ore to the autoclave



Meaning: If this approach was applied from the start, ie several years earlier, there would be more gold produced from the same assets.

Stochastic schedule - More ore, larger pit



ALL IMPROVEMENTS ARE DUE TO:

Managing Technical Risk from materials mined

and at the same time

Capitalizing on the Synergies between all parts of the mining complex

The End

Thanks are in order to our

COSMO Industry Members

AngloGold Ashanti, Barrick, BHP, De Beers, Kinross, Newmont Goldcorp, Vale

And
Funding Agencies

Canada Research Chairs / Chaires de recherche du Canada

NSERC CRSNG